PROBLEM 1. Let an $n$-PDA be a PDA with $n$ stacks.

(a) Why is a 1-PDA more powerful than a 0-PDA?

(b) What word would you use to prove that the language $L := \{a^n b^n c^n : n \geq 0\}$ is not context free. Say $p$ is a pumping length for a posited PDA. No explanation necessary.

(c) Prove that a 2-PDA more powerful than a 1-PDA. Give a specific example, and describe the relevant 2-PDA using numbered steps describing in English the action of the machine. You do not need to draw a state diagram.

It turns out that $n$-PDAs for $n \geq 2$ have the same power as Turing machines! That's something to think about after completing the rest of today's problems.

PROBLEM 2. Review the concept of an enumerator. Let $\Sigma = \{0, 1\}$. *Standard string order* on $\Sigma^*$ means ordering strings in increasing length and ordering strings of the same length in *lexicographic order* taking $0 < 1$:

$$\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, 101, 110, 111, 0000, \ldots$$

Show that a language $L$ is Turing decidable if and only if some enumerator enumerates the language in standard string order.

PROBLEM 3.

(a) Draw the state machine for a TM that takes as input a word of the form $w\#v$ where $w, v \in \{0, 1\}^*$, and outputs $w\llcorner\#v$. Notate your transitions as in the text. For example,

$$q_1 \xrightarrow{0 \to 1, R} q_2$$

means that if the machine is in state $q_1$ and the head reads a 0, then it writes a 1, moves the head to the right, and moves to state $q_2$. There is also a shorthand notation,

$$q_1 \xrightarrow{0 \to R} q_2$$

means that if the machine is in state $q_1$ and the head reads a 0, then it writes a 0, moves the head to the right, and moves to state $q_2$.

(b) Trace the set of configurations your TM moves through starting with the input 001#10. Use the notation from the book, e.g., $11q_201$ means machine is in state $q_2$ and the head is reading a 0.

PROBLEM 4.

(a) List all binary strings of length at most 4 in standard order: $\varepsilon, 0, 1, 00, 01, \ldots$

(b) Describe an enumerator that prints words in $\Sigma = \{0, 1\}$ in standard string order. Your enumerator is a TM but with a print command which, when called, prints the contents of the tape from the beginning to the first blank cell. Your solution should again be a list of steps with descriptions in English. You do not need to produce a state diagram.

PROBLEM 5. Let $D$ be the set of polynomials $p$ in $n$ variables and integer coefficients such that $p(x_1, \ldots, x_n) = 0$ has a solution $a = (a_1, \ldots, a_n) \in \mathbb{Z}^n$. Why is $D$ Turing recognizable? You need to describe a Turing machine that halts (after a finite number of steps) if and only if there is a solution. Recall that Hilbert's tenth question is, essentially, whether $D$ is Turing *decidable*.