PROBLEM 1.

(a) Discuss how you would prove that the intersection of a context-free language and a regular language is a context-free language. Afterwards, give a written explanation using fewer than three sentences.

(b) Explain how to prove that the set of binary strings with equal numbers of 0s and 1s but containing no substring of the form 0100 or 1001 is context free. Detailed descriptions of machines are not required.

PROBLEM 2. Review the precise statement of the pumping lemma for context-free languages.

Let $\Sigma = \{1, 2, 3, 4\}$ and L be the set of words $w \in \Sigma^*$ such that the number of 1s in w equals the number of 2s in w, and the number of 3s in w equals the number of 4s. Prove that L is not a CFL.

PROBLEM 3. Let $\Sigma = \{0, 1\}$, and let P be the set of words that are palindromes with an equal number of 0s and 1s: $P = \{ww^R : w \in \Sigma^* \text{and the number of } 0s \text{ equals the number of 1s in } w\}$. Prove that P is not a CFL.

PROBLEM 4.

(a) What is the formal definition of a Turing machine?

(b) Can a Turing machine ever write the blank symbol ␣on its tape?

(c) Can the tape alphabet Γ ever be the same as the input alphabet Σ?

(d) After executing a transition, is it ever possible that the head of a Turing machine is over the same cell it started in?

(e) Can a Turing machine contain just a single state?

PROBLEM 5. Give a high-level description with numbered steps of a Turing machine that recognizes the palindrome language of Problem 3.

PROBLEM 6. (Bonus) Consider a Turing machine that cannot write over its input. That is, whatever length of string that the input is on cannot be changed, but can be read as normal (and the rest of the tape can be changed as normal). Show that Turing machines of this type can recognize only regular languages.