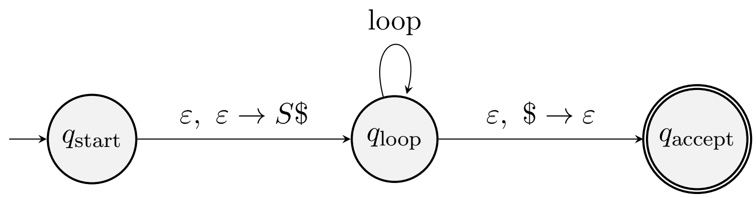
Problem 1. Let A be the following CFG from our text (Example 2.25):

$$S \to aTb|b$$
$$T \to Ta|\varepsilon.$$

The text describes the construction for the following PDA that recognizes $L(A)$:



The loop has the following transitions:

$$\varepsilon,\ S \to aTb|b$$
$$\varepsilon,\ T \to Ta|\varepsilon$$
$$a,\ a \to \varepsilon$$
$$b,\ b \to \varepsilon.$$

Here, for instance, the notation $\varepsilon,\ S \to aTb|b$ is shorthand for two transitions: $\varepsilon,\ S \to aTb$ and $\varepsilon,\ S \to b$. Also, for example, the transition $\varepsilon,\ S \to aTb$, means that if S is at the top of the stack, one can pop S and push aTb without reading any input.

(a) Describe $L(A)$.

(b) Show how the PDA accepts $aaab$ one step at a time. List the new state, the transition, and the stack. The first step is

transition	q_{start}	$\xrightarrow{\varepsilon,\varepsilon\to S\$}$	q_{loop}
stack	empty	$\longrightarrow$	$S\$$

Problem 2. Let $\Sigma = \{0, 1\}$, and consider the language $L = \{w : w \text{ contains at least three 1s}\}$.

(a) Find a regular expression for L.

(b) Convert that regular expression into an NFA recognizing L.

(c) Find a CFG for the language.

(d) Convert your CFG into a PDA using the algorithm in the text (as in Problem 1).

Problem 3. Let $\Sigma = \{0, 1\}$, and consider the language $L = \{0^m 1^n : m \neq n\}$.

(a) Find a CFG generating L using at most three variables.

(b) Convert your CFG into a PDA using the algorithm in the text (as in Problem 1).

(c) Is L regular? Prove or disprove.

Problem 4. (Bonus) Our goal in this problem is to show that the representation of objects can affect whether or not a given set can be recognized by a machine. Consider a set A

of natural numbers. Let $B_k(A)$ be the set of strings that represent numbers from A in base k (with no leading zeros). For example, if $A = \{3, 5\}$ then $B_2(A) = \{11, 101\}$ and $B_3(A) = \{10, 12\}$. We can think of $B_k(A)$ as a language with a k-symbol alphabet. Give a set A for which $B_2(A)$ is regular but $B_3(A)$ is not (and prove it).