Problem 1. For each of the following, prove either that the language is regular or that it is not regular. In all cases $\Sigma = \{0, 1\}$.

(a) $L_1 = \{w : w \text{ contains an equal number of 0s and 1s}\}$.

(b) $L_2 = \{1^k w : w \in \Sigma^*, k \geq 1, \text{ the number of 1s in } w \text{ is at least } k\}$.

(c) $L_3 = \{1^k w : w \in \Sigma^*, k \geq 1, \text{ the number of 1s in } w \text{ is at most } k\}$.

Problem 2. Consider the context-free grammar G

$$S \to RR$$
$$R \to 0R1 \mid \varepsilon.$$

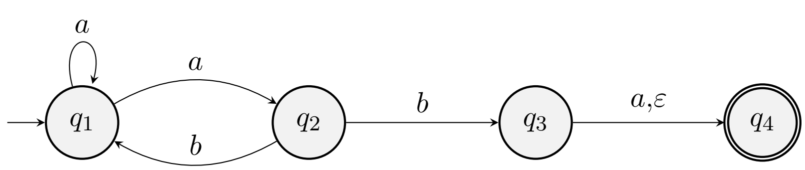
(a) Which of the following are in $L(G)$? For each that is in $L(G)$, give a left-most derivation making one substitution at each step.
   (i) 001101
   (ii) 000111.
   (iii) 1010.
   (iv) ε.

(b) Succinctly describe the language $L(G)$. Explain your reasoning.

(c) Is $L(G)$ regular? Provide a proof.

Problem 3. Let A be the language of all words in $\Sigma = \{0, 1\}$ that contain the same number of occurrences of 01 as occurrences of 10. Is A regular? Provide a proof.

Problem 4. Let A be the NFA pictured below:



(a) Create a CFG G such that $L(A) = L(G)$.

(b) Give a derivation of the word *aabab* making one substitution at each step.

Problem 5. Let $P = \{0^q : q \text{ is a prime number}\}$. Prove or disprove that P is regular.

Problem 6. (Bonus) Our goal in this problem is to show that the representation of objects can affect whether or not a given set can be recognized by a machine. Consider a set A of natural numbers. Let $B_k(A)$ be the set of strings that represent numbers from A in base k (with no leading zeros). For example, if $A = \{3, 5\}$, then $B_2(A) = \{11, 101\}$ and $B_3(A) = \{10, 12\}$. We can think of $B_k(A)$ as a language with a k-symbol alphabet. Give a set A for which $B_2(A)$ is regular but $B_3(A)$ is not (and prove it).