Math 387 Group problems, Friday Week 2

For the following, let $\Sigma = \{0, 1\}$.

PROBLEM 1. Find CFGs for the following languages: .

(a) $L = \{w : w \text{ has odd length and its middle symbol is } 0\}$.

(b) $L = \{w : w \text{ has twice as many 0s as 1s}\}$.

Solution.

(a)

$$S \to 0S0|0S1|1S0|1S1|0$$

(b)

$$S \to T|\varepsilon$$
$$T \to TT1|T1T|1TT|0.$$

PROBLEM 2.

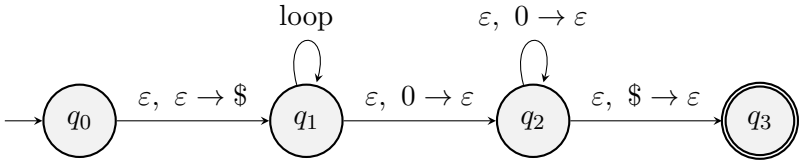
(a) Show that context free languages are closed under the union, concatenation, and star operations. (By the way: they are not closed under intersection and complementation, in general.) For notation, let A and B be context free grammars with start states S_A and S_B, respectively. Describe context free grammars generating the languages $L(A) \cup L(B)$, $L(A)L(B)$, and $L(A)^*$. You may assume that the variables for A and B are distinct.

(b) With notation as above, give an example that shows $L(A)^*$ is not necessarily generated by the context free grammar obtained from A by adding the rule $S_A \to S_A S_A$ to the starting rules for A.

PROBLEM 3. Draw the state diagram of a PDA that accepts the language

$$L = \{w : w \text{ contains more 0s than 1s}\}.$$

Solution. A possible PDA is



where the loop has transitions:

$$0, \$ \to 0\$$$
$$0, 0 \to 00$$
$$0, 1 \to \varepsilon$$

$$1, \$ \to 1\$$$
$$1, 0 \to \varepsilon$$
$$1, 1 \to 11.$$

PROBLEM 4. (Bonus) Create a CFG that generates the language $L = \{xy : |x| = |y|, x \neq y\}$.