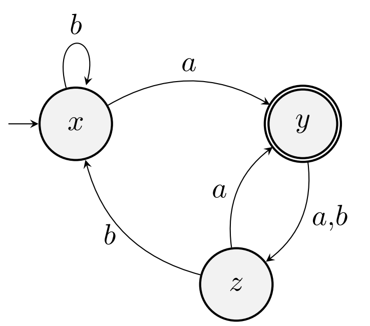
Math 387 Group problems, Monday Week 1

Except for Exercise 8, you are asked to create DFAs (deterministic finite automata) for the listed languages on the alphabet $\Sigma = \{0, 1\}$.

1. Σ^*, the collection of all words, including the empty word ε.
2. The empty language, $\emptyset$.
3. $\{\varepsilon\}$.
4. $\{\varepsilon, 110\}$.
5. Words of length 4.
6. Words with length a multiple of 4.
7. Nonempty words whose beginning and ending characters differ.
8. Consider the following DFA.

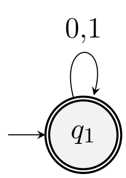


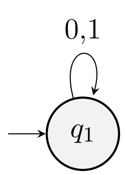
   (i) Give a formal description $(Q, \Sigma, \delta, q_0, F)$ for this language, including a table defining δ.

   (ii) What language is recognized by this DFA?

9. Construct DFAs A and B such that $L(A)$ is the language of all words with length divisible by 2 and $L(B)$ is the language of all words with length divisible by 3. Use the Cartesian product construction from the text to create a DFA C such that $L(C) = L(A) \cup L(B)$, labeling the states accordingly.
10. Words with length divisible by either 2 or 3 but not both.
11. Words with the same number of 0s as 1s.

Solution.

1. 

2. 

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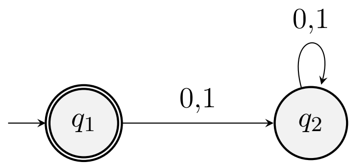

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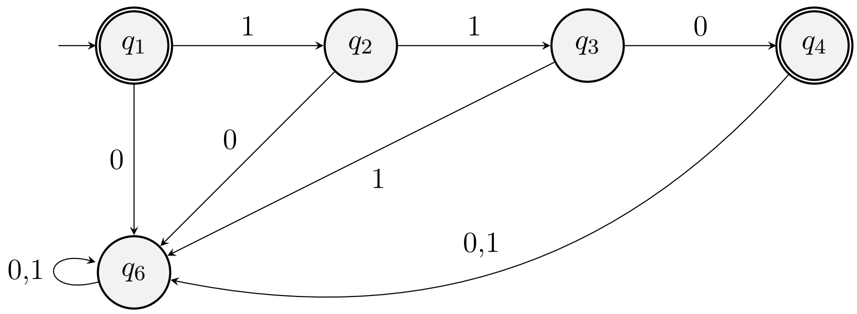

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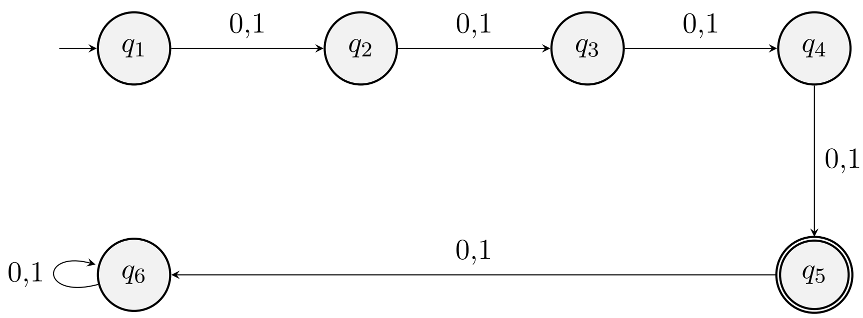

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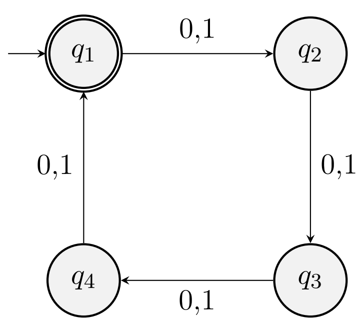

7. To appear in homework.

8. (i) $Q = \{x, y, z\}$, $\Sigma = \{a, b\}$, $q_0 = x$, and $F = \{x\}$. The transition function $\delta\colon \Sigma \times Q \to Q$ is given by the table

	a	b
x	y	x
y	z	z
z	y	x

(ii) This is the language of all strings ending with an odd number of as.

9. To appear in homework.

10. To appear in homework.

11. None exists. (We will prove this later using the pumping lemma.)