Interview with Richard P. Stanley

The Wonderful World of Combinatorics

Interviewed by Shaoshi Chen

Richard Stanley has been Professor Emeritus of Mathematics at MIT since January 2018. He received his BS in mathematics from Caltech in 1966, and his PhD in mathematics from Harvard University in 1971, under the direction of Gian-Carlo Rota. He joined the MIT faculty in applied mathematics in 1973, and became a professor in 1979. Professor Stanley's research concerns problems in algebraic and enumerative combinatorics. Professor Stanley's distinctions include the SIAM George Pólya Prize in applied combinatorics in 1975, a Guggenheim fellowship in 1983, the Leroy P. Steele Prize for Mathematical Exposition in 2001, the Rolf Schock Prize in Mathematics in 2003, and the Leroy P. Steele Prize for Lifetime Achievement in 2022. Professor Stanley was the inaugural Levinson Professorship Chair of Mathematics at MIT, 2000–2010. He was appointed Senior Scholar at the Clay Mathematics Institute in 2004, and received an Honorary Doctorate from the University of Waterloo. In 2007, he received an Honorary Professorship from Nankai University. He is a Fellow of the American Academy of Arts & Sciences (1988) and a Member of the National Academy of Sciences (1995). He was an invited speaker at ICM1983 and a plenary speaker at ICM2006.

Figure 1. Richard Stanley in the memorial room of Professor Wen-tsun Wu.

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Figure 2. Richard Stanley with the author in the memorial room of Professor Wen-tsun Wu.

SC (Shaoshi Chen): Thank you very much for accepting this interview. We would like to ask you some questions about you and your mathematics. Let's start with: When did you realize that you loved mathematics and wanted to be a mathematician?

RS (Richard Stanley): I think it was roughly at the age of thirteen when I was in the ninth grade. I moved to Savannah, Georgia. There was another student in my class (named Irvin Asher) who seemed to be doing something that I thought was very advanced mathematics. I was very interested in what he was doing and became inspired to learn more about mathematics. That was the start. I then began to go to the library and check out all of the popular math books in order to learn as much math as I could.

SC: Did anyone help you understand the books? Or did you just study by yourself?

RS: At that time in Savannah there were really no mathematicians, no good universities or anything similar, and I was on my own.

SC: When did you make the decision that you wanted to be a mathematician?

RS: Maybe in high school when I was thirteen or fourteen years old. I thought I would go into astronomy or physics, and after the experience with this classmate, then math became a possibility. By the time I graduated from high school, I was pretty sure that I preferred mathematics. In college, I took a lot of physics courses to see which I liked better, but I never really changed my mind about math.

SC: During your undergraduate study, was there a teacher who influenced you very much? Who made you understand or learn more about mathematics?

RS: I remember there were many good teachers at Caltech. Maybe the one with the most influence on me was Marshall Hall. He is a very well-known group theorist, but he was also interested in combinatorics and wrote one of the few books on combinatorics at that time. I was mainly interested in group theory from his influence, not combinatorics. In fact, I was not interested in taking a combinatorics course from Marshall Hall because I did not consider combinatorics to be a serious subject! I wanted to go to Harvard University to be a graduate student and work with the group theorist Richard Brauer. Finite group theory is like a combination of combinatorics and algebra, so you can see I had some nascent combinatorial interests. But when I graduated from Caltech I was thinking only of algebra, or perhaps number theory. Incidentally, I took a graduate group theory course from Marshall Hall, and in the same class was Michael Aschbacher who became very famous later for his role in the classification of finite simple groups. I should also mention that it was Marshall Hall who got me a summer job at JPL (the Jet Propulsion Laboratory, operated by Caltech for NASA, and responsible for the missions of unmanned extraterrestrial spacecraft). I spent around seven consecutive summers working in the coding theory group at JPL.

SC: Maybe I will ask a stupid question, namely, I know there is another group theorist named Phillip Hall. So is there any connection between those two Hall's?

RS: People always ask this question. The answer is that there is no connection.

SC: I know that the famous combinatorist Gian-Carlo Rota was your PhD supervisor. Can you say something about Rota and how he supervised you or how he influenced you during your graduate study?

RS: Yes, certainly his biggest influence was to get me to work in combinatorics. I never thought that it was really a serious subject, but I got interested in some combinatorial problems mainly from my job at JPL. I asked people at Harvard (where I was a graduate student) about my problems. They suggested that I should see Gian-Carlo Rota at MIT. Rota was very enthusiastic, suggesting all kinds of combinatorics that I should learn and convinced me that I should work with him in combinatorics. But I actually ended up doing most of the work on my own, just talking to him about other topics, not directly related to my research.

SC: What is combinatorics in your opinion?

RS: People are always asking me about that. The answer depends on their level of mathematics. Combinatorics deals with discrete structures. There are many different questions you can ask: what are the best ways or most efficient ways of arranging things and organizing them? How many ways are there to do it? How easy is it to find these solutions or prove that solutions exist or connect them with other things? It's very basic for much of mathematics to arrange discrete objects according to certain rules in the best and most elegant ways, or to see how many ways (or approximately how many ways, if an exact answer cannot be found) there are to do it, and to understand the structure formed by all these arrangements. Thus I think the problem of how to arrange discrete objects in certain interesting and elegant ways, and to understand how many of them there are and how they are related to each other, are basic questions in combinatorics.

SC: *On your MIT homepage, it says that your main research area is algebraic combinatorics. Can you give a brief introduction to algebraic combinatorics?*

RS: Well, it is basically the way in which combinatorics and algebra are connected with each other. I like both subjects: combinatorics and algebra—they are very natural to me. In algebraic combinatorics, you can go in either direction. You can apply algebra to combinatorics by embedding combinatorial objects into algebraic structures and using algebra to obtain information about the combinatorics objects. Or you can do it in the other direction, to understand the algebraic objects by looking at their combinatorial properties, and applying combinatorial reasoning. People were doing this since Euler, Jacobi, Cayley, and others. Of special importance to algebraic combinatorics today is the work of Frobenius, Young, Schur, and others on the representation theory of the symmetric group. These people never really thought of algebraic combinatorics as a separate subject, but they were very well-qualified to find some connections between algebra and combinatorics. The idea, mainly due to Rota, to try to develop algebraic combinatorics (also geometric combinatorics) in a systematic way really appealed to me.

SC: *Recently, I was reading the AMS book* The Mathematical Legacy of Richard P. Stanley, *which is in celebration of your 70th birthday. In this book, you wrote a very nice note on all your publications, briefly discussing how your papers started and how you finished them. You mentioned the "wishful thinking proof technique." I was curious about how this technique works.*

RS: Yes, this is the idea that when you're trying to prove something, you think about what is the best possible situation that could be true so that you could prove it or at least make further progress. It's a kind of wishful thinking that if only this were true, then I would have a chance of proving it. There's no reason to believe it's true at first, and most of the time it doesn't work. But several times in my career it actually worked amazingly well. For instance, a complicated power series in several variables arose in the theory of reduced decompositions of permutations. I had no idea on how to get any information about it unless it happened to be a symmetric function. It was just wishful thinking that it might be a symmetric function; I didn't have any reason to believe that it should be. I did some computations and saw that it seemed to be true. Once it seems to be true, then you can try to prove it and use it. I think it's a good idea when doing research to try to think about the best thing that could happen so you can make further progress, and then see if it works.

SC: *I think it's connected to your mathematical intuition. Before you really do the rigorous proof, you have intuition to believe it's true.*

RS: I call it wishful thinking because it is just a hope that might work. The key difference between wishful thinking and other kinds of conjecturing and guessing in mathematics is that in wishful thinking, you don't have any reason for believing your wish might come true. Intuition is not yet involved, but a good knowledge of possible tools and techniques is essential.

SC: *Many of your papers arise from some questions asked by non-mathematicians or mathematicians who are not doing combinatorics. Your answers to these questions increase the impact of combinatorics on other topics because it impresses people that combinatorics is a powerful tool to solve some questions that they want to answer. Can you say something about how combinatorics is connected to other areas, similar to how you already talked about the connection between algebra and combinatorics, according to your understanding?*

RS: Many subjects are connected with combinatorics. In the past, people worked on their problems and usually found the solutions by doing some combinatorial computations. They just leave it at that and think that's the answer. But combinatorics can actually push you much further. Solving combinatorial problems that arise in other areas gives you all kinds of insights into these areas. For instance in topology, which has concepts like tori, spheres, and ways of putting on handles and twisting space, you can easily imagine that some combinatorial structures are involved. Biology is another example that is full of combinatorial considerations, from phylogenetic trees to the genetic code to protein folding. The question is how to make this intuition rigorous and precise and related to serious combinatorics. I think that almost any branch of mathematics, as well as many other areas, has some connection to combinatorics. You could try to do some serious mathematics by explaining these connections in a precise way.

SC: You have written two classical books on enumerative combinatorics, Enumerative Combinatorics, *volumes 1 and 2 (known as* EC1 *and* EC2*) which now are perhaps the most influential books in combinatorics. When did you start this writing project?*

RS: It was in 1979, I was visiting UCSD (University of California at San Diego) when Pierre Leroux was visiting there from Montreal. We decided that it would be a good idea to write a book on enumerative combinatorics. I would write the text and he would make the problems. That was the original plan. But after I started to work on it, I realized that I wanted to do the whole thing by myself. I had a strong idea about how to do the problems and in many cases their solutions. So that was the start in 1979, and it took seven years before the first volume came out. That volume almost doubled in size in a second edition published in 2012. The second volume was published in 1999.

SC: In these books there are many interesting exercises which come from different papers. How did you collect so many interesting exercises? How did you organize them?

RS: Even before I started writing the book, maybe just after I got my PhD, I thought it would be a good idea to write down any kind of facts that I thought interesting or possibly useful. As you know there were no personal computers back then, so I got a lot of 3x5 index cards. I just wrote down some interesting fact on each card and put them in a file box. I still have them at my MIT office. When I started writing up exercises for my book, I used these cards as a basis. Any time I came across something interesting from reading, going to lectures, etc., I would just write it down on an index card. Thus this was my way to collect exercises.

SC: It's a very good habit. Could you talk about some of your work in graph theory and your opinion on the importance of the theory of graphs and networks?

RS: Well, concerning my own work in graph theory, I think it is purely algebraic and enumerative, and not too closely related to all these applications in network theory, operations research, etc. As an example of my work in graph theory, I earlier gave a talk in your institute about the chromatic polynomials of graphs. They are a special case of characteristic polynomials of hyperplane arrangements. They're connected with Mobius functions and partially ordered sets. I found all these connections very interesting. What could you say about chromatic polynomials or other polynomials arising from combinatorics? One special topic is about the reciprocity that arises when a polynomial is defined to have some nice combinatorial meaning for positive integers, and then turns out to have a nice meaning for negative integers. In a rather indirect way I was led to

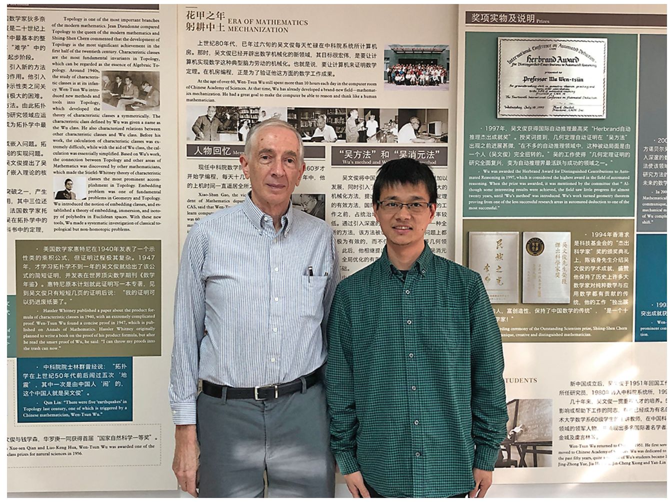

Figure 3. Richard Stanley with the author in the memorial room of Professor Wen-tsun Wu, in front of the bulletin board about how Wu promoted Computer Mathematics in China.

ask whether something like this happens with chromatic polynomials. That was one motivation for me to work in the area of graph theory.

SC: Nowadays we use computers every day and everywhere. Can you say something about the influence of computers on combinatorics, or on mathematics in general?

RS: Well, for just one thing, you can do experiments with a computer far beyond what you can do by hand. Sometimes you make discoveries that you just never could see by hand. In order to see a pattern in some data, you might have to work up to n=16, but by hand you can only go up to n=4. There are many examples like that. For myself, I use computers as a way to generate data for making conjectures. But as you know some people use computers to prove new theorems, like proving the four-color conjecture or Kepler's conjecture. I don't really go in that direction, but certainly these are very interesting developments which are different from just generating a lot of data and trying to make sense of it.

SC: I think you have visited China many times. When was the first time?

RS: In 1986, I attended a graph theory conference at Shandong University in Jinan. After the meeting I spent a few days in Beijing and visited Beijing University.

SC: Can you tell us some stories about your collaborations with Chinese mathematicians?

RS: Well, it started out with Bill Chen, my mathematical brother, that is, we have the same thesis adviser (Gian-Carlo

Figure 4. In the coffee room of Academy of Mathematics and Systems Science, CAS: Richard Stanley (sitting next to Atsuko Kida) was discussing the e-positivity conjecture with Dun Qiu, Arthur L.B.Yang, and Philip B. Zhang.

Rota). When Bill was a graduate student at MIT, I was a professor and taught him some classes. We started our collaboration then. Subsequently I had some Chinese PhD students. I didn't write joint papers with them when they were students (I like to have my students work as independently as possible), but I certainly collaborated with them on their thesis research. I guess you know Fu Liu, my Chinese student with whom I have most collaborated. I had very good collaborations with her resulting in two published papers. My Chinese graduate students (including students who grew up in Taiwan) and their degree dates are Bo-Yin Yang (1991), Wungkum Fong (2000), Fu Liu (2006), Jingbin Yin (2009), and Nan Li (2013). Other Chinese mathematicians I have worked with include Ruoxia (Rosena) Du, Yinghui Wang, Yuping (Eva) Deng, Huafei (Catherine) Yan, Xiaoying (Ellen) Qu, Xingmei (Sabrina) Pang, Beifang Chen, Xiaomei Chen, Xueshan (Teresa) Li, Lili Mu, Wuxing (Tommy) Cai, and Guoliang (David) Wang. Often I would first meet them in China at some meeting, and they would ask if they could visit MIT. I would make the arrangements, and then we would start the collaboration.

SC: *Yes, it's very helpful that people can go to MIT so that they can communicate with leading scholars, and thereby enlarge their research area. Up to now, you have supervised 60 PhD students in total. Many of them have become famous like Ira Gessel and Thomas Lam. Do you have any advice for young researchers or PhD students, especially in combinatorics?*

RS: Of course, giving some general advice to any students is not so easy, but I think it's very good that the students learn how to generate their own research problems or questions. Always keep your eyes open to something that is interesting, and if you think there's some way that you might be able to make a contribution, you shouldn't be afraid and say "that's not my area" or "people have already worked on this." You should, as long as you have some kind of "reasonable" idea, go ahead and pursue it. You need to have a bit of judgment about what is reasonable. For instance, I would not recommend working on the Riemann hypothesis unless you really think you have a new idea. When someone gives a talk and mentions some problem, if you find it interesting and even if it's not your exact area, I would say keep it in mind, go for it. Even if you don't get anywhere, you should try to remember everything, because you never know whether someday you might find something that you can use to eventually make progress or even solve the problem.

SC: *My last question. Which open problem in combinatorics is your favorite one? I think you have many in your mind. And you would like to see the solutions in the future?*

RS: Well, I could name two. The first one is definitely the g-conjecture for spheres (or certain more general simplicial complexes called Gorenstein). What can be the number of faces of each dimension of a triangulation of a sphere? Lou Billera, Carl Lee, and I solved this problem for simplicial convex polytopes in 1979. Since 1979, the g-conjecture is probably the biggest open problem in the area of the combinatorics of simplicial complexes. There has been some recent progress on this problem, in particular, by Karim Adiprasito, who posted a proof of the g-conjecture for spheres on the arXiv (see `https://arxiv.org/abs/1812.10454`), but so far no one has read the whole proof. It will be very exciting if it's correct.[1]

Another open problem I especially like is the e-positivity conjecture for certain chromatic symmetric functions, because it's connected with so many other areas like Kazhdan-Lusztig theory and Hessenberg varieties. I think there's all kinds of deep mathematics going on behind this conjecture, and I would really like to see what this looks like.

Credits

All figures are courtesy of the author.

[1] *In an email to me on March 17, 2022, Richard asked me to add a footnote here: "After this interview, Adiprasito's proof was carefully checked and is now accepted as correct."*