

Extra time: go over HW.

$$n \in \mathbb{N}$$

## Young Diagrams

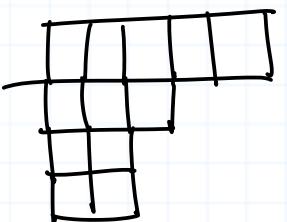
partition of  $n$ :  $(\lambda_1, \lambda_2, \dots)$  where  $\overset{\textcircled{1}}{\lambda_i} \in \mathbb{N}$ ,  $\overset{\textcircled{2}}{\lambda_1 \geq \lambda_2 \geq \dots}$ ,  $\overset{\textcircled{3}}{\sum_{i \geq 1} \lambda_i = n}$   
 notation  $\lambda \vdash n$ .

Examples

$$5 \ 3 \ 2 \ 2$$

Partition of 12 w/

4 parts



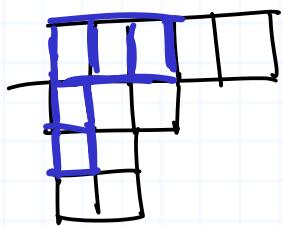
$$\begin{aligned}\lambda_1 &= 5 \\ \lambda_2 &= 3 \\ \lambda_3 &= 2 \\ \lambda_4 &= 2\end{aligned}$$

corresponding Young diagram

$$\text{Officially, } (5, 3, 2, 2, 0, 0, 0, \dots)$$

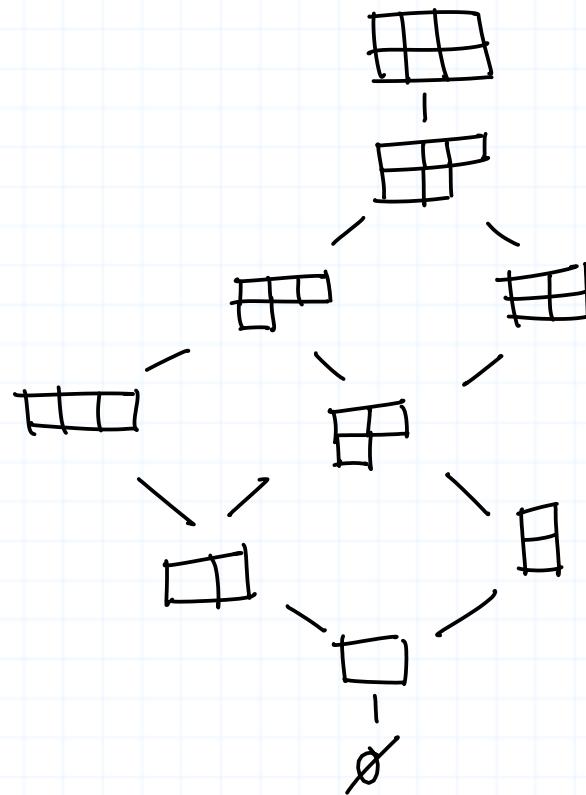
Partial order on partitions:  $\lambda \leq_M \mu$  if  $\lambda_i \leq \mu_i \quad \forall i$ .

$$\text{So } 311 \leq 5322$$



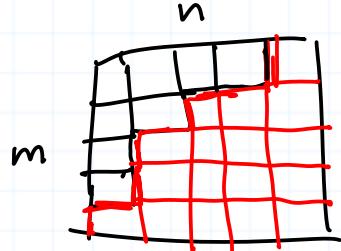
Poset of interest:  $L(m,n)$  = partitions with at most  $m$  parts  
and with each part at most  $n$ .

Example  $L(2,3)$

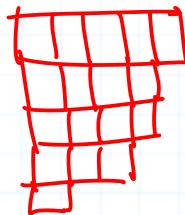


Goal:  $L(m,n)$  graded (with  $\text{rk}(\lambda) = |\lambda| := \sum_{i \geq 0} \lambda_i$ ),  
 rank  $m n$ , rank-symmetric, unimodal, Sperner.

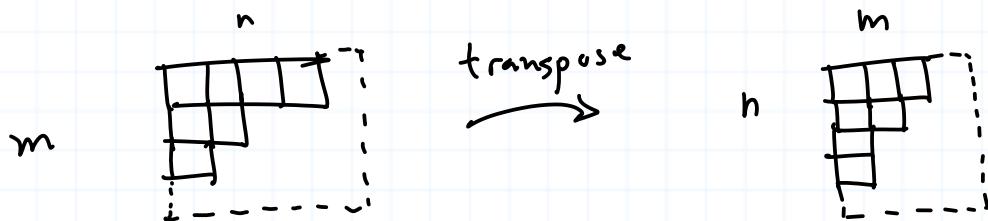
rank symmetry:



4211 has complement 54431



Note relationships between  $L(m,n)$  and  $L(n,m)$ :



Proposition 6.3.

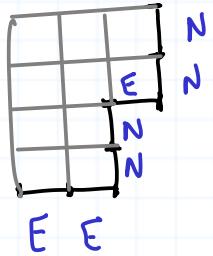
$$\# L(m,n) = \binom{m+n}{n}$$

Idea: bijection between strings of length  $m+n$  in  $N$  (= north)

and E (=east) with m N's and n E's.

Example

EEENNENN



Example

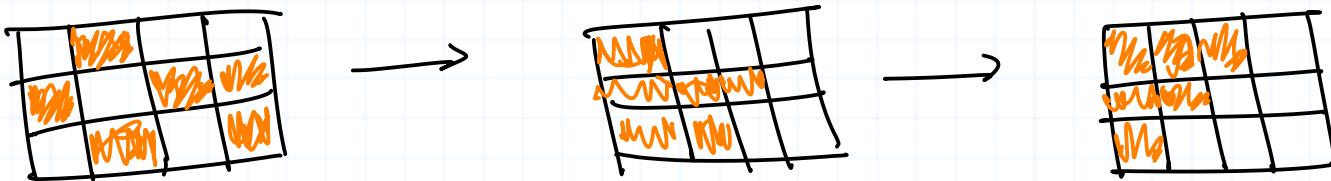


in  $L(3,5)$  : ENNENEEE

□

To show  $L(m,n)$  is Sperner and rank-unimodal, we show it's isomorphic to a quotient of the Boolean poset  $B_{mn}$ .

Consider all permutations of the boxes of the  $m \times n$  rectangle which arise from moves consisting of (1) permuting the boxes in a row and (2) permuting rows. This gives an action of the wreath group  $\mathfrak{S}_m \wr \mathfrak{S}_n$  on  $B_{mn}$  and hence on all subsets of  $B_{mn}$ .



In each orbit there is a unique Young diagram.

This gives the isomorphism of partially ordered sets.