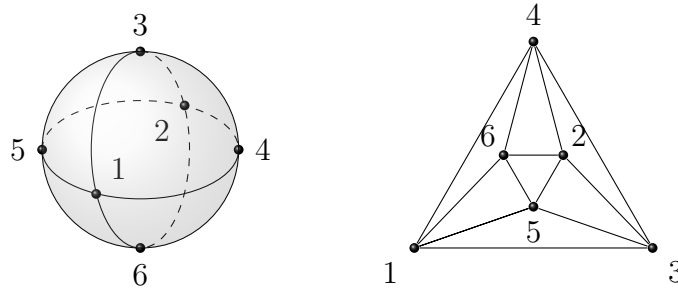


Math 372 Homework for Wednesday, Week 10

PROBLEM 1. (The boundary of a boundary is empty.) Show that the following holds for the boundary mappings in the chain complex for a simplicial complex:  $\partial_i \circ \partial_{i+1} = 0$ , for each  $i$ . This relation is often abbreviated as  $\partial^2 = 0$ .

PROBLEM 2. The left side of the figure below shows a simplicial complex drawn on a sphere. On the right side is the same simplicial complex projected onto the face  $\overline{134}$ . It has eight two-dimensional facets, counting  $\overline{134}$ .



- (a) Find matrices for all nonzero  $\partial_i$  with respect to our standard bases (lexicographic ordering of faces).
- (b) Compute the rank and nullity of each nonzero  $\partial_i$  and use them to compute all of the Betti numbers for the complex.
- (c) Find a generator for  $\tilde{H}_2$  with geometrical significance.

PROBLEM 3. Let  $\Delta$  be the simplicial complex with facets

$$\overline{1234}, \overline{125}, \overline{135}, \overline{235}, \overline{36}, \overline{56}, \overline{67}, \overline{89}, \overline{8*}, \overline{9*}.$$

where  $*$  = 10 (for typesetting purposes). Compute all of the Betti numbers without doing a calculation. However, you should explain your reasoning.

PROBLEM 4. Let  $\Delta$  be a simplicial complex, let  $f_i$  be the number of faces of dimension  $i$ . For example,  $f_{-1} = 1$ , and  $f_0$  is the number of vertices. Let  $\tilde{\beta}_i$  be the  $i$ -th Betti number of  $\Delta$ . Show that

$$\tilde{\beta}_0 - \tilde{\beta}_1 + \tilde{\beta}_2 - \cdots = -f_{-1} + f_0 - f_1 + \cdots .$$

PROBLEM 5. Let  $B_3$  be the boolean poset of all subsets of  $\{1, 2, 3\}$ .

- (a) Describe the order complex  $\Delta(B_3)$  as a geometric object. Attempt a drawing of this object. To do this, first draw the Hasse diagram for  $B_3$  with the elements labeled with letters  $a$  through  $h$  starting with  $a = \emptyset$ ,  $b = \bar{1}$ , and so on, to  $h = \overline{123}$  so that each succeeding letter is the label for the next subset in lexicographical ordering.
- (b) What are the Betti numbers for  $\Delta(B_3)$ ? (You should be able to compute these numbers from your answer to part (a) without performing a calculation.)