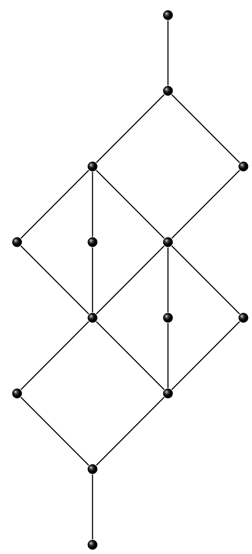
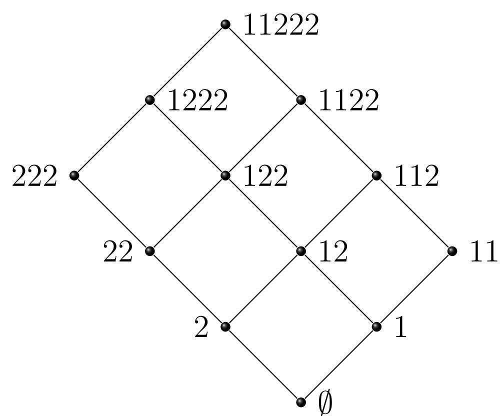
Math 372 Homework for Wednesday, Week 5

PROBLEM 1. Consider the poset P whose Hasse diagram is pictured by



Prove there is no subgroup H of the symmetric group $\mathfrak{S}_7$ such that $P \simeq B_7/H$.

PROBLEM 2. Let M be a finite multiset, say with a_i i's for $1 \leq i \leq k$. Let B_M denote the poset of all submultisets of M, ordered by multiset inclusion. For instance, the figure below [see text] illustrates the case $a_1 = 2, a_2 = 3$. Use Theorem 5.8 to show that B_M is rank-symmetric, rank-unimodal, and Sperner. (There are other ways to do this problem, but you are asked to use Theorem 5.8. Thus you need to find a subgroup G of $\mathfrak{S}_n$ for suitable n for which $B_M \simeq B_n/G$.)



Problem 3. Find an explicit order matching $\mu\colon L(2,n)_i \to L(2,n)_{i+1}$ for $i < n$.

Problem 4.

(a) Draw the Hasse diagram for the poset $L(2,4)$.

(b) From the Hasse diagram, find the rank-generating function $P(2,4) = \sum_{i\geq 0} p_i(2,4)\, q^i$ for $L(2,4)$

(c) Verify Theorem 6.6 in the text by computing a certain q-binomial coefficient.

Problem 5.

(a) Make a conjecture for the number of maximal chains in $L(2,n)$ of maximal length, i.e., maximal chains starting at the unique maximal element (n,n) of the poset and ending at the unique minimal element $\emptyset$.

(b) Prove your conjecture.