Math 372 Homework 1

Problem 1.

(a) Let G be the graph with vertices v_1, v_2, v_3, v_4, obtained from the complete graph by deleting the edge $\{v_1, v_3\}$. Use the corollaries to Theorem 1.1 to find the number of closed walks of length ℓ. Hint: to help factor the characteristic polynomial, note that both 0 and -1 are roots.

(b) Describe these closed walks for $\ell = 0, 1, 2, 3$.

Problem 2. Show that the trace of a square matrix is the sum of its eigenvalues. (Hint: use the Jordan canonical form and standard properties of the trace.)

Problem 3. (Chapter 1, Exercise 2.) Suppose that the graph G has 15 vertices and that the number of closed walks of length in G is $8^\ell + 2 \cdot 3^\ell + 3 \cdot (-1)^\ell + (-6)^\ell + 5$ for all $\ell \geq 1$. Let G' be the graph obtained from G by adding a loop at each vertex (in addition to whatever loops are already there). How many closed walks of length ℓ are there in G' ? (Use linear algebraic techniques. You'll need Lemma 1.7 and the trick we used to find the eigenvalues for the adjacency matrix for the complete graph from the matrix of all 1s.)

Problem 4. (Chapter 1, Exercise 4.) Let $r, s \geq 1$. The complete bipartite graph K_{rs} has vertices $u_1, u_2, \ldots, u_r, v_1, v_2, \ldots, v_s$, with one edge between each u_i and v_j (so rs edges in all).

(a) By purely combinatorial reasoning, compute the number of closed walks of length ℓ in K_{rs}.

(b) Deduce from (a) the eigenvalues of K_{rs}.