

Math 361 Quiz topics

Let R be a ring, and let L/K be an extension of fields.

- What does it mean to say $p \in R$ is *prime*? a *unit*?, *irreducible*?
 - What does it mean to say that R is an *integral domain*.
 - Let R be a integral domain, and let $a, b, c \in R$. Suppose that $a \neq 0$ and $ab = ac$. Prove that $b = c$. (Warning: since R is not a field, we can't assume that a has a multiplicative inverse. Also, where do we use the fact that R is a integral domain?)
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- What does it mean to say that $I \subseteq R$ is an ideal?
- What does it mean to say that $I \subseteq R$ is finitely-generated?
- What does it mean to say that $I \subseteq R$ is a principal ideal?
- What is the general relationship between prime elements and irreducible elements of R ? What if R is a PID?
- Let K be a field. State the division algorithm for $K[x]$.
- Let K be a field. Explain at a high level (i.e., assuming standard results from algebra) why $K[x]$ is a UFD.

Let L/K be an extension of fields.

- What does it mean to say $\alpha \in L$ is *algebraic* over K ?
 - If $\alpha \in L$ is algebraic over K , what is the minimal polynomial for α over K ?
 - Suppose $\alpha \in L$ is algebraic over K , and let p be the minimal polynomial for α over K . What is $[L : K]$ in terms of p ?
 - If $[L : K] < \infty$ and $\alpha \in L$, is it necessarily true that α is algebraic over K ?
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- What is the set of *algebraic numbers* \mathbb{A} ? What is the set of *algebraic integers* \mathfrak{D} ?
- Let M be a finitely generated R -module. What does it mean for M to be *free*. Up to isomorphism, what do finitely generated R -modules look like?
- Let $A \subseteq B$ be an extension of domains, and let $\alpha \in B$.

- What does it mean for α to be *integral* over A ?
- Prove that if there exists a finitely generated A -module $M \subset B$ such that $\alpha M \subseteq M$, then α is integral over A . (Recall our theorem that gives two conditions that are equivalent to α being integral over A .)
- What does Gauss's lemma say about factorization of polynomials with integer coefficients?
- Why is an algebraic integer always algebraic over \mathbb{Q} ? How can you characterize an algebraic integer in terms of its minimal polynomial?
- What is a *number field*? What is the *ring of integers* in a number field?
- State the primitive element theorem.
- Let $d \neq 0, 1$ be a square-free integer. Identify the ring of integers in $\mathbb{Q}(\sqrt{d})$?