Math 361 Quiz topics

Let R be a ring, and let L/K be an extension of fields.

- What does it mean to say $p \in R$ is *prime*? a *unit*?, *irreducible*?
- What does it mean to say that R is an *integral domain*.
- Let R be a integral domain, and let $a, b, c \in R$. Suppose that $a \neq 0$ and $ab = ac$. Prove that $b = c$. (Warning: since R is not a field, we can't assume that a has a multiplicative inverse. Also, where do we use the fact that R is a integral domain?)

- What does it mean to say that $I \subseteq R$ is an ideal?
- What does it mean to say that $I \subseteq R$ is finitely-generated?
- What does it mean to say that $I \subseteq R$ is a principal ideal?
- What is the general relationship between prime elements and irreducible elements of R? What if R is a PID?
- Let K be a field. State the division algorithm for $K[x]$.
- Let K be a field. Explain at a high level (i.e., assuming standard results from algebra) why $K[x]$ is a UFD.

Let L/K be an extension of fields.

- What does it mean to say $\alpha \in L$ is *algebraic* over K?
- If $\alpha \in L$ is algebraic over K, what is the minimal polynomial for α over K?
- Suppose $\alpha \in L$ is algebraic over K, and let p be the minimal polynomial for α over K. What is $[L : K]$ in terms of p?
- If $[L : K] < \infty$ and $\alpha \in L$, is it necessarily true that α is algebraic over K?