## Math 316 Homework for Friday, Week 8

PROBLEM 1. Let R be a finite domain. Prove that R is a field. (Hint: given  $0 \neq r \in R$ , consider the multiplication mapping  $m_r \colon R \to R$  given by  $m_r(s) = rs$  for  $s \in R$ . Now use the properties that R is a domain and R is finite.)

PROBLEM 2. Show  $r^2 + 5s^2 = 2$  has no solutions  $r, s \in \mathbb{Q}$ .

PROBLEM 3. Find some solutions to  $x^2 + y^2 = z^3$  for which x, y, z are pairwise relatively prime and  $2 < x \le y$ . (We already know everything about the case x = 2 from class.)

Bonus: In class on Wednesday, Week 6, we saw that  $x^2 + 4 = z^3$  has exactly four integer solutions. Does the reasoning involved in finding those solutions tell us anything about solutions to  $x^2 + y^2 = z^3$ ?