PROBLEM 1. The Chow ring of $\mathbb{G}_1\mathbb{P}^4$.

(a) List all the Schubert classes for $\mathbb{G}_1\mathbb{P}^4$. For each class, state the codimension, give both the a-notation and λ-notation, and describe the associated Schubert condition. For example, one line of your list will be

codim	class	condition
3	$(1,3)=\{2,1\}$	to lie in a solid a meet a line lying in that solid

a "solid" being a 3-dimensional linear subspace.

(b) Make the 10×10 multiplication table for the Chow ring of $A^*(\mathbb{G}_1\mathbb{P}^4)$.

(c) Find m so that there will be a finite number of lines meeting m given lines, in general. Then find the number of lines meeting m general lines.

(d) How many lines will meet 6 planes, in general?

(e) What is the degree of $\mathbb{G}_1\mathbb{P}^4 \subset \mathbb{P}^9$ (Plücker embedding)? (Hint: The *degree* is the number of times a generic linear space of complementary dimension intersects the set. Since $\mathbb{G}_1\mathbb{P}^4$ has dimension $(r+1)(n-r)=6$, i.e., codimension 3, the degree is given by the number of times a general solid (3-dimensional subspace, codimension 6) in $\mathbb{P}^9$ meets the Grassmannian. To create this solid, note that the Schubert class of codimension 1 is given by intersecting the Grassmannian with a hyperplane.)

(f) Create your own enumerative problem that can be answered using this Chow ring subject to the condition that the solution be a positive integer.

(a)

codim	class	condition
6	$(0,1) = \{3,3\}$	to be a given line
0	$(3,4) = \{0,0\}$	no condition

(b)

	1									$\yng(3,3)$
1	1									$\yng(3,3)$
										0
										0
										0
										0
										0
										0
										0
										0
$\yng(3,3)$	$\yng(3,3)$	0	0	0	0	0	0	0	0	0

PROBLEM 2. (See Example 14.29 in our text.) Give geometric explanations for the following calculations in $A^\bullet\mathbb{G}_1\mathbb{P}^3$:

(a) $(0,3)^2 = (1,2)^2 = (0,1)$;

(b) $(1,3)(0,3) = (1,3)(1,2) = (0,2)$;

(c) $(0,3)(1,2) = 0$;

(d) $(1,3)^2 = (0,3) + (1,2)$. (For this problem, use that fact that for some problems in Schubert calculus, it is OK to allow the conditions to degenerate. For instance, what if the two lines involved in $(1,3)^2$ were allowed to intersect? For a fuller explanation of this phenomenon, see Schubert calculus according to Schubert, by Felice Ronga.)