MATH 341 TOPICS IN GEOMETRY
COURSE INFORMATION & SYLLABUS

FALL 2020

Place:	Physics 122
Time:	MWF 13:10–14:00 P.M.
Instructor:	David Perkinson (`davidp@reed.edu`)
Office Hours:	See our course Moodle page
Textbook:	*Manifolds*, notes by Perkinson and Xu
	(optional) *Vector Analysis*, by Klaus Jänich
Website:	`people.reed.edu/~davidp/341/`

Course summary. This is a course in differentiable manifolds. Just as vector spaces generalize the linear structure of Euclidean space, manifolds generalize the differential structure. In both the case of vector spaces and manifolds, coordinates are useful, but secondary. Manifolds can be embedded in Euclidean space, just as vector spaces can be embedded as linear subspaces. Familiar examples of embeddings of manifolds include spheres and tori.

After "recalling" some background in topology and vector calculus, we will give the definition of a manifold. Our first goal is to define the *tangent space* to the manifold at a given point, which we will do in three equivalent ways. (Try to image how one might do that for an object that does not have a fixed set of coordinates and is not embedded in space!)

Next, we introduce some multilinear algebra: tensors, dual spaces, and exterior and symmetry products of vector spaces. Our main application will be to make useful constructions using the tangent space at each point in a manifold.

We gather all of the tangent spaces of a manifold together (one for each point) in a structure called the *tangent bundle*, which is, itself, a manifold. The tangent bundle and other related *vector bundles*, constructed via multilinear algebra, will allow us to do calculus on a manifold.

We will shoot for the following goals:

- » **Stokes' theorem** for a manifold: $\int_M d\omega = \int_{\partial M} \omega$. To make sense of this, we will need to introduce the idea of an *orientation* of a manifold, *differential forms* (which will be sections of the top exterior power of the cotangent bundle), and manifolds with *boundary*.
- » De Rham cohomology.
- » Poincare duality for a Riemannian manifold. (Riemannian manifolds are manifolds with an extra structure—a *metric*—that allows measurement of distance on the manifold. The metric is itself a section of a certain vector bundle.
- » Interesting examples: toric varieties and Grassmannians. The cohomology for a Grassmannian has a ring structure (i.e., a binary multiplication as well as the usual addition). The resulting *Schubert calculus* allows one to compute answers to questions such as: Fix four lines in 3-space in general position. How many lines can be draw to intersect all four lines?

This course would be good preparation for graduate school in mathematics.

Learning outcomes. After taking this course, students will have a firm understanding of the basic theory of abstract differentiable manifolds at level suitable for a stiff graduate-level course in the subject. They will also have considerable experience working as part of a small group to solve mathematical problems and practice at communicating mathematical ideas verbally and in writing.

Distribution requirements. This course can be used towards your Group III, "Natural, Mathematical, and Psychological Science," requirement. It accomplishes the following goals for the group:

» Use and evaluate quantitative data or modeling, or use logical/mathematical reasoning to evaluate, test, or prove statements.
» Given a problem or question, formulate a hypothesis or conjecture, and design an experiment, collect data or use mathematical reasoning to test or validate it.

This course **does not** satisfy the "primary data collection and analysis" requirement.

Texts. The course will use notes based on the text *Vector Analysis*, by Klaus Jänich. The Jänich text is recommended but not required. The notes will be available as a PDF file on the course website which will be updated as the semester proceeds. (If you find typos or have suggestions for improving the text, please let me know!)

Homework. Homework assignments will be posted on our course homepage and will be due each Friday via Gradescope.[1] Excellent solutions take many forms, but they all have the following characteristics:

» they use complete sentences, even when formulas or symbols are involved;
» they are written as explanations for other students in the course; in particular, they fully explain all of their reasoning and do not assume that the reader will fill in details;
» when graphical reasoning is called for, they include large, carefully drawn and labeled diagrams;
» they are neatly typeset using the LaTeX document preparation system. A guide to LaTeX resources is available on the course homepage.

I reserve the right to not accept late homework. If health or family matters might impede the timely completion of your homework, please contact me as early as possible.

Feedback. You will receive timely feedback on your homework via Gradescope. Most homework problems will be graded on a five-point scale (5 = perfect; 4 = minor mistake; 3 = major mistake, right idea; 2 = significant idea; 1 = attempted, 0 = none of the above). *The quality of your writing will be taken into account.* If your answer is incorrect, this will be reflected in the score, and there will also be a comment indicating where things went wrong with your solution. You are strongly encouraged to engage with this comment, understand your error, and try to come up with a correct solution.

Collaboration. You are permitted and encouraged to work with your peers on homework problems. It is best practice to cite those with whom you worked, and you must write up solutions independently. **Duplicated solutions will not be accepted and constitute a violation of the Honor Principle.**

Quizzes and Exams. We will have a short review quiz at the beginning of each Wednesday class, a midterm exam, and a final exam. It is to be determined whether the exams will be in-class or take-home.

Grades. Your grade will be based on the quality of your homework, the quizzes, the exams, and class participation.

[1] Gradescope is an online homework submission and evaluation platform. You are likely to already be enrolled in our Gradescope class. If not, you will be able to enroll using a link+code provided on our Moodle page.

Academic honesty: As noted above, for homework you should write your own solutions and disclose your collaborators. The internet is a great source of information about mathematics; you are welcome to search for information about the material of the course online, but you should not search for solutions to specific problems in the homework. **Copying solutions from fellows students or from the Internet is an Honor Principle violation and will result in an academic misconduct report.**

Help. There are a number of resources you can access for help with this course's content. Everyone is welcome and encouraged to attend my **office hours**. They are an opportunity to clarify difficult material and also delve deeper into topics that interest you. Further, every Reed student is entitled to one hour of free **individual tutoring** per week. Use the tutoring app in IRIS to arrange to work with a student tutor.

Technology: The use of electronic devices (computers, cell phones, tablets, etc.) is not allowed in the classroom without my authorization. Browsing the internet, answering your email, and texting during class is rude—it disrupts learning. It distracts your classmates and your instructor. Talk to me if you have a specific reason for needing to use technology (for example, note-taking).

Academic accommodations. If you have a documented disability requiring academic accommodation, please have Disability & Accessibility Resources (DAR) provide a letter during the first week of classes. We can then discuss your accommodations. I cannot provide accommodations after an assignment has been turned in or within 24 hours of an exam. If you believe you have an undocumented disability and that accommodations would ensure equal access to your Reed education, I would be happy to help you contact DAR.