Math 341 Quiz topics for Week 5

- Let (U, h) be a chart at p on a manifold M. Define the tangent vector $(\partial/\partial x_i)_p$ with respect to (U, h) as an element of $T_p^{\text{geo}}M$, as an element of $T_p^{\text{alg}}M$, and as an element of $T_p^{\text{phy}}M$.
- Let $f\colon M \to N$ be a mapping of manifolds, and let $p \in M$. Define $df_p\colon T_pM \to T_{f(p)}N$, the differential mapping at p, under our three interpretations of tangent space at p.
- Let e_1, e_2, e_3 be the standard basis for $\mathbb{R}^3$, and let $u = (1, 2, 3)$, $v = (4, 0, 1)$, and $w = (2, -1, -1)$ in $\mathbb{R}^3$. Write the following in terms of standard bases for the appropriate tensor spaces:

  (a) $u \otimes v + 3v \otimes w \in \mathbb{R}^3 \otimes \mathbb{R}^3$.

  (b) $u \cdot v + 3v \cdot w \in \text{Sym}^2\mathbb{R}^3$.

  (c) $u \wedge v + 3v \wedge w \in \Lambda^2\mathbb{R}^3$.

- What is the universal property of the tensor product? symmetric product? exterior product?
- What is the dual of a real vector space V?
- Let $v_1, \ldots, v_n$ be a basis for a vector space V. What is the dual basis for V^*?
- Let $f\colon \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $f(x, y) = (2x + y, 5x, x - y)$, and let $\phi\colon \mathbb{R}^3 \to \mathbb{R}$ be defined by $\phi(u, v, w) = u + 2v + w$. Then $\phi \in (\mathbb{R}^3)^*$. What is the pullback $f^*\phi \in (\mathbb{R}^2)^*$.
- What is an alternating ℓ-form? Explain why an alternating ℓ-form on a vector space V can be identified with an element of $(\Lambda^\ell V)^*$, and hence, with an element of $\Lambda^\ell V^*$.