Math 341 Reading Questions for Monday, Week 3

In the following, refer to Figure 9 in our text which exhibits the mappings Φ_1, Φ_2, and Φ_3 relating our three notions of tangent space.

QUESTION 1. Let $p = (1,2) \in M = \mathbb{R}^2$. Consider the curve $\alpha(t) = (1,2) + t(3,4)$ in M passing through p at time $t = 0$, and the function $f(x,y) = x^2 + xy$ on M. Let $\alpha\colon \mathcal{E}_p \to \mathbb{R}$ be the derivation associated with $[\alpha] \in T_p^{\text{geom}}$, i.e., $v_\alpha = \Phi_1([\alpha])$.

What is $v_\alpha(f)$?

QUESTION 2. Let p, M, α, and v_α be as in Question 1. Consider the chart (M,h) where $h(x,y) = (3x+y, 2x+y)$. Let $\tilde{v}$ be the element of T_p^{phy} corresponding to v_α. What is $\tilde{v}(M,h)$.

Solutions appear on the next page.

Question 1: 16.

Question 2: $(13, 10)$.