Math 322 Homework 3

PROBLEM 1. Solve each of the following differential equations. Your solution should have the form $y =$ etc. In other words, I'm looking for an explicit solution. Don't leave answers with complex numbers, e.g., use sines and cosines rather than e^{it}.

1. $y'' - 2y' + y = 2\cos(t) + 4e^{3t}, \quad y(0) = y'(0) = 1.$
2. $ty' + 5y - t^5 y^2 = 0, \quad y(1) = 1.$
3. $y'' + 2y' + 3y = 5 + 3t.$
4. $y' = yt/(t^2 + 1), \quad y(0) = -3.$
5. $y'' - 6yy' = 0, \quad y(0) = 2,\ y'(0) = 9.$
6. $y'' - 6yy' = 0, \quad y(0) = 2,\ y'(0) = 0.$
7. $D^2(D + 1)^3(D^2 + 2D + 2)^2 y = 0.$
8. $y^{(4)} - 16y = 0.$
9. $y'' = -2(y')^2, \quad y(0) = 1 = y'(0) = 1.$

PROBLEM 2. Let $A \in M_n(F)$, and let r_i be the i-th row of A for $i = 1 \ldots n$. Let $\ell = \max\{|r_i| : 1 \le i \le n\}$, the maximum length of a row of A. Prove $\|A\| \le \ell\sqrt{n}$. (Strive to find an elegant solution that does not involve referencing the elements of A by name. You can do everything using just the notation introduced in the statement of the problem. Consider $|Ax|^2$ for $|x| \le 1$, and use Cauchy-Schwarz. On the other hand, an ugly solution is still a solution.)