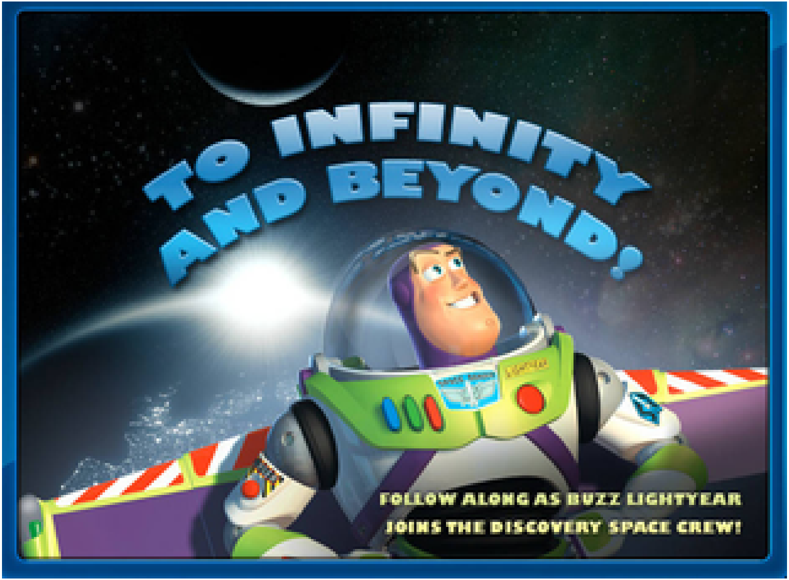
Math 322

April 18, 2022

Global phase portraits



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y' &= 5xy - 5
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Critical points of the system, including critical points at ∞.

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&= y\left(\left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2 - 1\right) - x\left(5\left(\frac{x}{z}\right)\left(\frac{y}{z}\right) - 5\right) \\
&= \frac{x^2y + y^3 - yz^2 - 5x^2y + 5xz^2}{z^2}
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Clear denominators and set $z = 0$:

$$
-4x^2y + y^3 = y(-4x^2 + y^2) = 0.
$$

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Projection to $x = 1$ plane:

$$u' = v^2\left(\frac{5u}{v^2} - 5 - u\left(\frac{1}{v^2} + \frac{u^2}{v^2} - 1\right)\right) = 4u - 5v^2 - u^3 + uv^2$$

$$v' = -v^3\left(\frac{1}{v^2} + \frac{u^2}{v^2} - 1\right) = -v - u^2 v + v^3.$$

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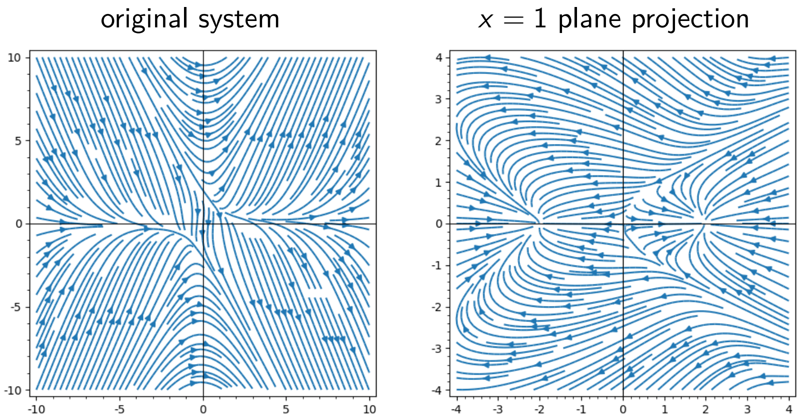
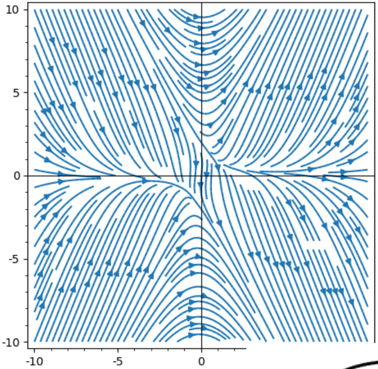
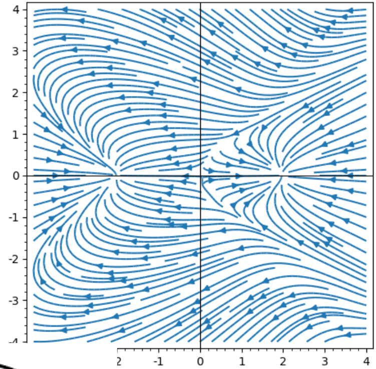
$$Jf(0,0) = \begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad J(\pm 2, 0) = \begin{pmatrix} -8 & 0 \\ 0 & -5 \end{pmatrix}.$$

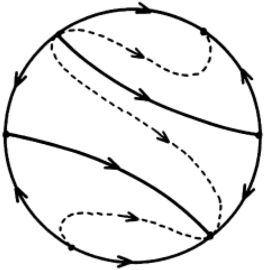

original system
$x = 1$ plane projection
10
5
0
-5
-10
4
3
2
1
-1
-2
-3
-4

original system

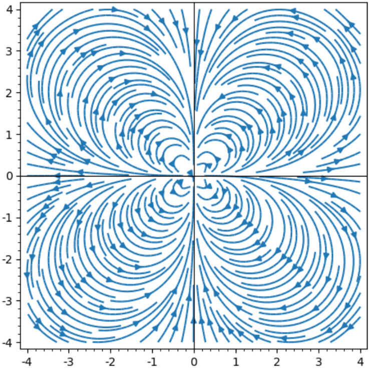


$x = 1$ plane projection





$$x' = x^3 - 3xy^2$$
$$y' = 3x^2y - y^3.$$



Points to check: $\pm(1, 0, 0)$ and $\pm(0, 1, 0)$.

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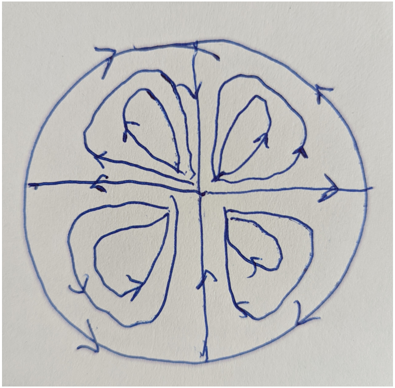
$$\begin{aligned} u' &= 2u + 2u^3 \\ v' &= -v + 3u^2 v \end{aligned}$$

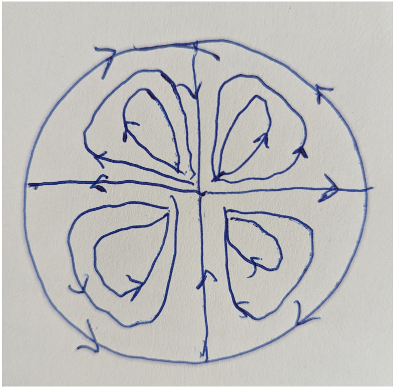
For the point $(0,1,0)$, project to the plane $y = 1$. The system is

$$\begin{aligned} u' &= -2u - 2u^3 \\ v' &= v - 3u^2 v \end{aligned}$$

Global phase portrait:

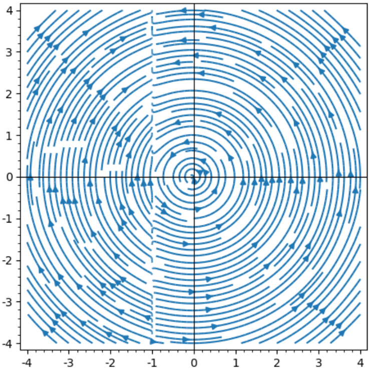


Global phase portrait:



Note that the sum of the indices for the corresponding flow on the sphere is 2, as is necessary.

$$x' = -y - xy$$
$$y' = x + x^2.$$



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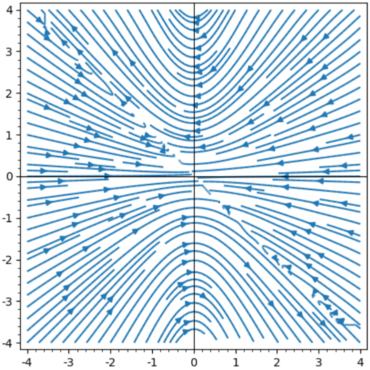
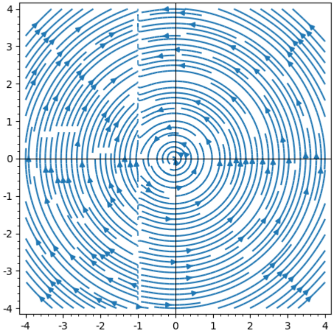
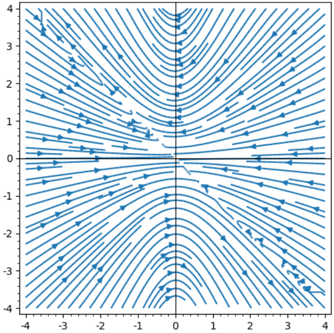
$$v' = -u^2 v - uv^2,$$




4
3
2
1
0
-1
-2
-3
-4
-4 -3 -2 -1 0 1 2 3 4


4
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