Math 322

April 11, 2022

Presentations

Presentation proposals due next Monday.

Index of a vector field on a manifold

- oriented 2-dimensional manifold (surface)
- vector field on S
- index of a critical point on a manifold

Poincaré-Hopf index theorem

Theorem. Let f be a vector field on a compact surface S of genus g, Suppose the critical points of f are $p_1, \ldots, p_k$. Then

$$\sum_{i=1}^{k} I_f(S) = 2 - 2g.$$

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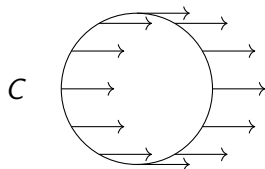
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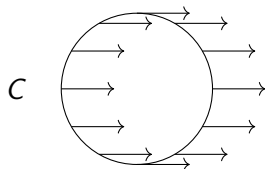
Proof. Case 1: $S = S^2$.

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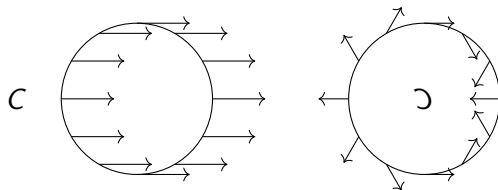
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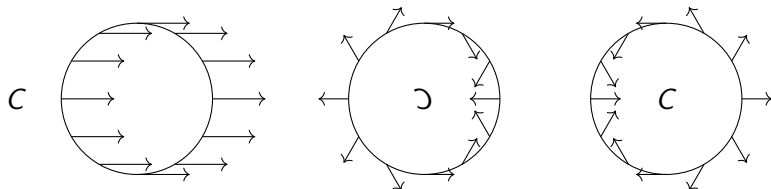
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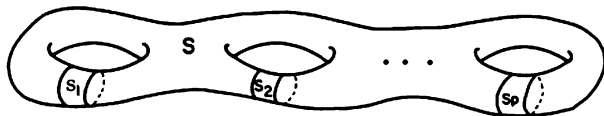
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