Math 322

February 11, 2022

Derivative of matrix exponential function

Lemma. (p. 16) Let $A \in M_n(F)$. Then

$$\frac{d}{dt} e^{At} = Ae^{At}.$$

Higher-order system

The Fundamental Theorem for Linear Systems

Theorem. Let $A \in M_n(F)$, and let $x_0 \in F^n$. The initial value problem

$$x' = Ax$$
$$x(0) = x_0$$

has the unique solution

$$x = e^{At}x_0.$$

Example

$$x_1' = x_2$$
$$x_2' = x_1$$

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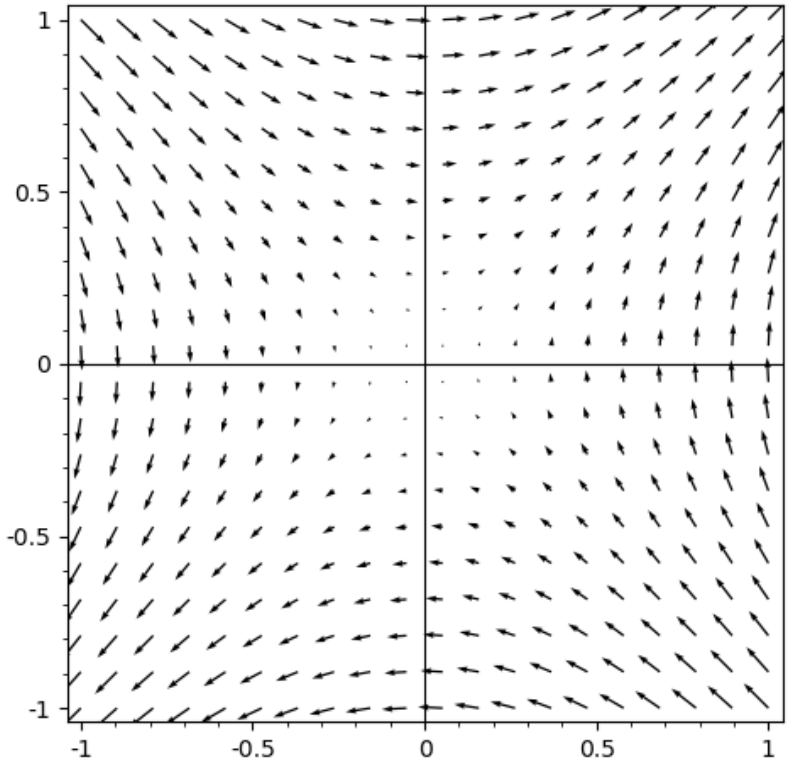

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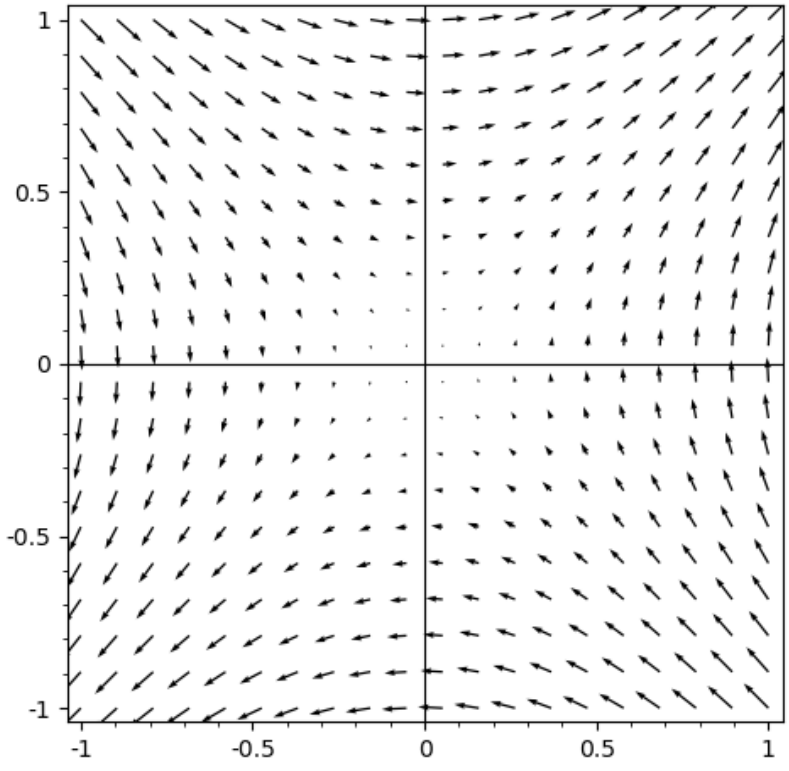
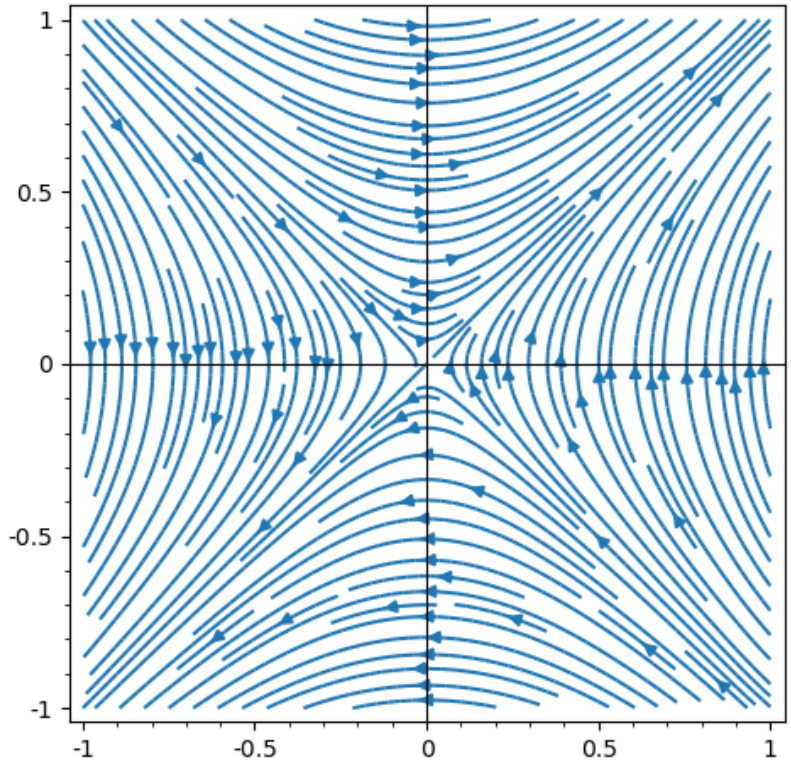
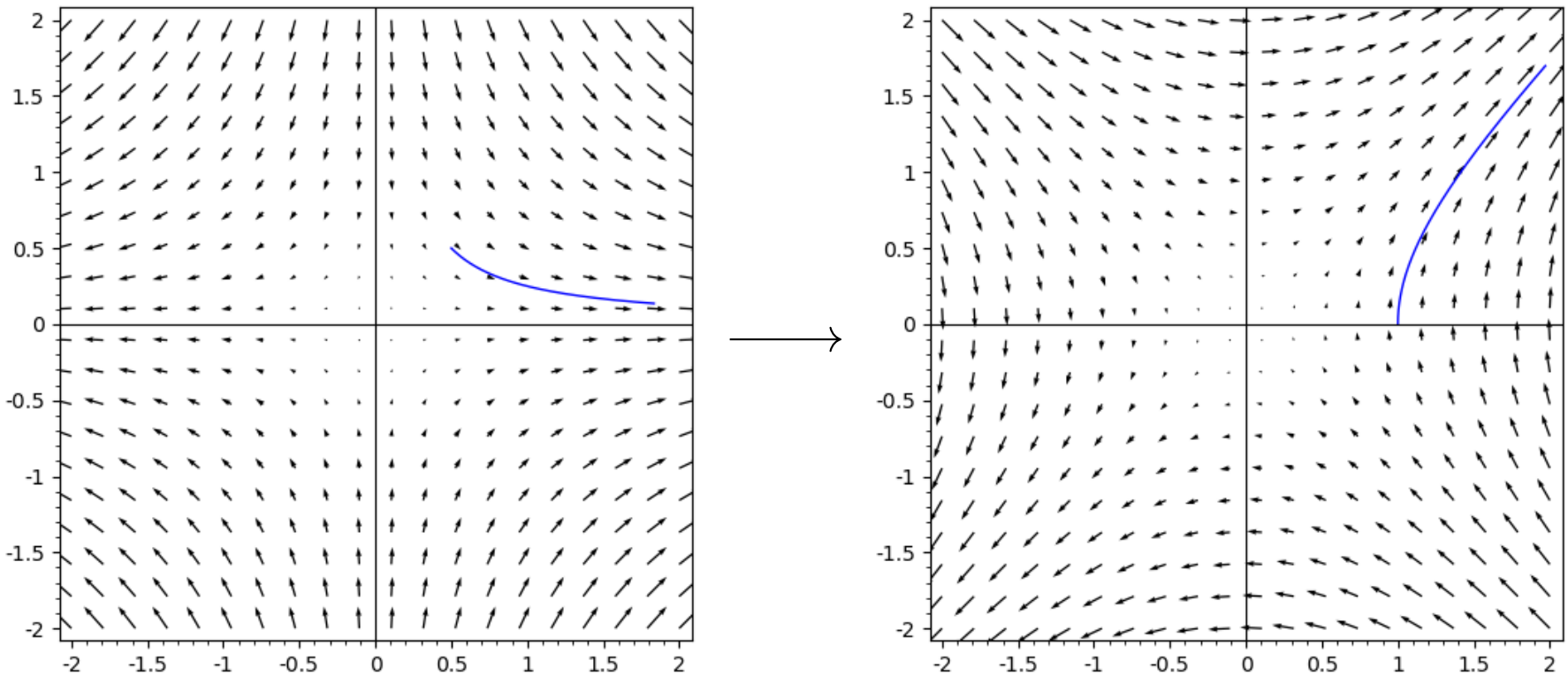




Example continued with initial condition $x(0) = (1, 0)$

$$P = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



$$y \to Py = x$$