Math 322

January 31, 2022

Announcements

- job talks

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- questions?

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Integrate:

$$e^{\int p(t)\,dt}y = \int e^{\int p(t)\,dt}q(t)\,dt,$$

Example

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Consider

$$\cos(t)\, y' + y = \sin(t)$$

with initial condition $y(0) = 1$.

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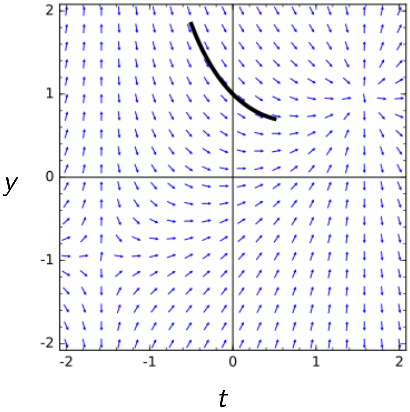
Solution: $y = \frac{\sec(t)+\tan(t)-t}{\sec(t)+\tan(t)} = 1 - \frac{t}{\sec(t)+\tan(t)}.$



III. B. Bernoulli-type first-order linear

$$\frac{dy}{dt} + p(t)y = q(t)y^m, \quad m \neq 1$$

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$$y' = \frac{2y}{t} - t^2y^2, \quad y(1) = -2.$$

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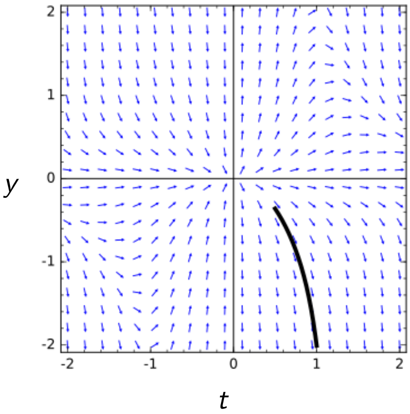
Solution: $\frac{t^2}{y} = \frac{1}{5}t^5 - \frac{7}{10} \quad \Rightarrow \quad y = \frac{10t^2}{2t^5 - 7}.$



IV. A. Linear homogeneous constant coefficients (LHCC)

Example.

$$y'' - y' - 6y = 0$$

with initial conditions $y(0) = 0$ and $y'(0) = 1$.

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  $$P(r) = r^2 - r - 6.$$

  Letting $D := \frac{d}{dt}$, our equation is

  $$P(D)y = 0.$$

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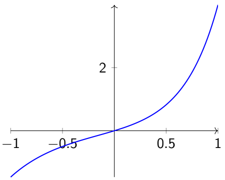

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The zeros of $P(r)$ determine basic solutions e^{rt}.

Initial conditions $y(t_0), \ldots, y^{(n-1)}(t_0)$ determine *unique* solution.