Math 322

February 4, 2022

Announcements

- Check your solutions, when possible

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- questions?

Method of undetermined coefficients example

$$y'' - 2y' + y = t\cos(3t)$$

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$$\begin{aligned} y'' - 2y' + y = &(-8a_0 - 2a_1 - 6b_0 + 6b_1 - (8a_1 + 6b_1)t)\cos(3t) \\ &+ (6a_0 - 6a_1 - 8b_0 - 2b_1 + (6a_1 - 8b_1)t)\sin(3t) \end{aligned}$$

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$$\begin{aligned} 0 &= -8a_0 - 2a_1 - 6b_0 + 6b_1 \\ 1 &= -8a_1 - 6b_1 \\ 0 &= 6a_0 - 6a_1 - 8b_0 - 2b_1 \\ 0 &= 6a_1 - 8b_1 \end{aligned}$$

Example, continued

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$$y_p = -\frac{1}{250}\left(13 + 20\,t\right)\cos\left(3\,t\right) - \frac{3}{250}\left(-3 + 5\,t\right)\sin\left(3\,t\right)$$

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$$y = y_h + y_p$$

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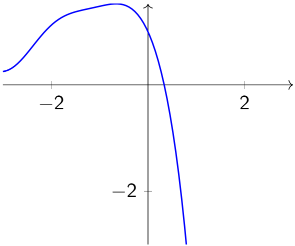
$$y = \frac{263}{250}\, e^{t} - \frac{77}{25}\, te^{t} - \frac{1}{250}\,(13 + 20\,t)\cos(3\,t) - \frac{3}{250}\,(-3 + 5\,t)\sin(3\,t)$$



VI. A. Second-order

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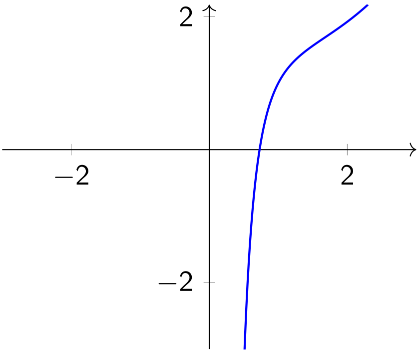
Solution:

$$y = \frac{1}{18}t^3 - \frac{11}{18}\,\frac{1}{t^3} + \frac{14}{9}.$$



VI. A. Second-order, example w/ different initial condition

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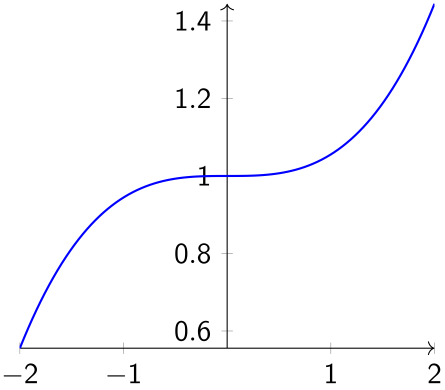
Solution: $y = 1 + \frac{1}{18}\,t^3$



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After finding v as a function of y, solve for y be integrating.

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Using the initial conditions, we get

$$\frac{1}{3}y^3 + 2y = -2 + 2t.$$

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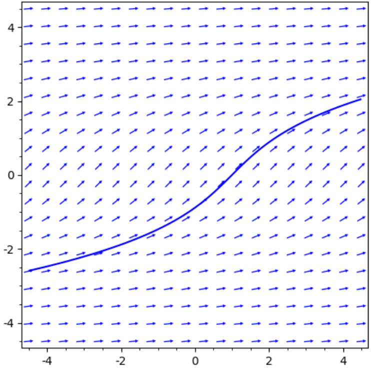

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Challenge. Solve

$$y'' + (y')^3 y = t.$$

with initial condition $y(0) = 1$ and $y'(0) = 0$.