Math 322

January 26, 2022

Announcements

- Math colloquium/job talk

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- LaTeX workshop: this Thursday, 7–8 p.m.

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- Practice problems.

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- Behavior as $P(t) \to K$?

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What happens as $t \to \infty$?

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Example. A state game commission releases 40 elk into a game refuge. After 5 years, the elk population is 104. The commission believes that the refuge can support no more than 4000 elk. Use a logistic model to predict the elk population in 15 years.

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$$P(15) = \frac{4000}{1 + 99e^{-0.194\cdot 15}} \approx 626.$$

Separable—homogeneity trick

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Trick: substitute $v = y/t$ to get separable equation:

$$y' = F\left(\frac{y}{t}\right) \Rightarrow v + tv' = F(v) \Rightarrow \frac{v'}{F(v) - v} = \frac{1}{t}$$

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Example.

$$y' = \frac{y^2 + 2yt}{t^2}$$

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Solution:

$$y = \frac{at^2}{1 - at}.$$

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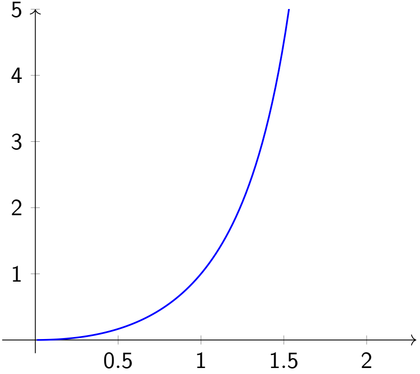

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Sage code:

```
sage: t = var('t')
sage: y = function('y')(t)
sage: desolve(diff(y,t)-(y^2+2*y*t)/t^2,y)
-(t^2 + t*y(t))/y(t) &=  _C
sage: desolve(diff(y,t)-(y^2+2*y*t)/t^2,y,ics=[1,1])
-(t^2 + t*y(t))/y(t) &=  -2
```