Math 111 Homework for Friday, Week 13

NOTE: You must show your work for credit on these problems.

PROBLEM 1. Use properties of exponents and the natural logarithm to solve for x:

(a) $\ln x = e$.

(b) $e^{x^2-1} - 1 = 0$.

(c) $27^x = \dfrac{9^{2x-1}}{3^{2x}}$.

(d) $\dfrac{1}{e^{-\ln x}} = 5$.

(e) $e^{\ln 2x} = 12$.

PROBLEM 2. Take the derivatives of the following functions with respect to x:

(a) $\ln(4x^2 + 3x + 1)$.

(b) $\ln e^x$.

(c) $\dfrac{1}{e^x + e^{-x}}$.

(d) $xe^{-2/x}$.

(e) $\ln \sqrt{x}$.

PROBLEM 3. Compute the following indefinite integrals (remembering to add $+c$):

(a) $\displaystyle\int \frac{6x^2}{x^3+5}\,dx$.

(b) $\displaystyle\int \frac{e^x}{1+e^x}\,dx$.

(c) $\int \ln(x)\, dx$ (hint: integration by parts with $u = \ln(x)$).

(d) $\int \frac{\ln x}{x}\, dx$.

(e) $\int \frac{\sin(x)}{1+\cos(x)}\, dx$.

PROBLEM 4. We know that $(e^x)' = e^x$ and $\ln'(x) = 1/x$. You can use these facts to find $(2^x)'$. Let $y = 2^x$. We want to compute y' (where the derivative is with respect to x). Taking logs, we get $\ln y = \ln(2^x)$. By a property of the logarithm, we have $\ln 2^x = x\ln(2)$. Hence,

$$\ln y = x \ln(2).$$

Use implicit differentiation to compute y' and express your solution solely in terms of x (i.e., if a y appears in your solution, replace it by 2^x).

PROBLEM 5. Compute the equation of the tangent line to the graph of $y = e^{-3x}$ at the point $(0, 1)$ in the form $y = mx + b$.