Math 111 Homework for Friday, Week 11

Please show your work for the following computations.

PROBLEM 1. Compute the following definite integrals:

(a) $\displaystyle\int_1^8 \sqrt{\frac{2}{x}}\, dx.$

(b) $\displaystyle\int_{-\pi/2}^{\pi/2} (2t + \cos(t))\, dt.$

PROBLEM 2. Compute the following indefinite integrals using u-substitutions. Be explicit about your substitution: $u = ?$, $du = ?$. Remember to add $+c$ to get the most general form for an antiderivative, and remember to express your solution in terms of the original variable (not u).

(a) $\displaystyle\int 2x(x^2 - 9)^3\, dx.$

(b) $\displaystyle\int t^2(t^3 + 5)^4\, dt.$

(c) $\displaystyle\int \sin^5(3\theta)\cos(3\theta)\, d\theta.$

(d) $\displaystyle\int -4x(1 - 2x^2)^{1/3}\, dx.$

(e) $\displaystyle\int \frac{x^3}{(1 + x^4)^2}\, dx.$

(f) $\displaystyle\int \frac{x^3}{\sqrt{1 + x^4}}\, dx.$

(g) $\displaystyle\int (y + 1)\sqrt{2 - y}\, dy.$

PROBLEM 3. Compute the following indefinite integrals by parts. Be explicit with your substitutions: $u = ?$, $dv = ?$ (and remember to add $+c$).

(a) $\displaystyle\int 3t\cos(2\pi t)\,dt.$

(b) $\displaystyle\int (4+3x)e^x\,dx.$

PROBLEM 4. We would like to prove that $\ln(xy) = \ln(x) + \ln(y)$ for all $x > 0$ and $y > 0$.

(a) As a warm up, compute the derivatives of the following functions using the chain rule and the fact that $(\ln(x))' = 1/x$.

  (i) $\ln(4x)$.

  (ii) $\ln(x^2 + x + 3)$.

(b) Now fix $y > 0$ and define a function of just the variable x by

$$f(x) = \ln(xy).$$

  (i) Prove that $f'(x) = 1/x$ using the chain rule. (Recall that y is a constant.)

  (ii) Since $f'(x) = (\ln(x))'$, these two functions must differ by some constant c:

$$f(x) = \ln(x) + c.$$

  Evaluate f at 1 to determine the value of c and establish the result we are trying to prove. (Recall the definition of f.)