Please show your work for the following computations.

**Problem 1.** Compute the following definite integrals:

(a) \( \int_1^8 \frac{\sqrt{2}}{x} \, dx \).

(b) \( \int_{-\pi/2}^{\pi/2} (2t + \cos(t)) \, dt \).

**Problem 2.** Compute the following indefinite integrals using \( u \)-substitutions. Be explicit about your substitution: \( u = ?, \ du = ?, \) Remember to add +c to get the most general form for an antiderivative, and remember to express your solution in terms of the original variable (not \( u \)).

(a) \( \int 2x(x^2 - 9)^3 \, dx \).

(b) \( \int t^2(t^3 + 5)^4 \, dt \).

(c) \( \int \sin^5(3\theta) \cos(3\theta) \, d\theta \).

(d) \( \int -4x(1 - 2x^2)^{1/3} \, dx \).

(e) \( \int \frac{x^3}{(1 + x^4)^2} \, dx \).

(f) \( \int \frac{x^3}{\sqrt{1 + x^4}} \, dx \).

(g) \( \int (y + 1)\sqrt{2 - y} \, dy \).

**Problem 3.** Compute the following indefinite integrals by parts. Be explicit with your substitutions: \( u = ?, \ dv = ? \) (and remember to add +c).
(a) \(\int 3t \cos(2\pi t) \, dt\).

(b) \(\int (4 + 3x)e^x \, dx\).

**Problem 4.** We would like to prove that \(\ln(xy) = \ln(x) + \ln(y)\) for all \(x > 0\) and \(y > 0\).

(a) As a warm up, compute the derivatives of the following functions using the chain rule and the fact that \((\ln(x))' = 1/x\).

(i) \(\ln(4x)\).

(ii) \(\ln(x^2 + x + 3)\).

(b) Now fix \(y > 0\) and define a function of just the variable \(x\) by

\[ f(x) = \ln(xy). \]

(i) Prove that \(f'(x) = 1/x\) using the chain rule. (Recall that \(y\) is a constant.)

(ii) Since \(f'(x) = (\ln(x))'\), these two functions must differ by some constant \(c\):

\[ f(x) = \ln(x) + c. \]

Evaluate \(f\) at 1 to determine the value of \(c\) and establish the result we are trying to prove. (Recall the definition of \(f\).)