Math 111 Homework for Friday, Week 10

Let f be a function defined on an interval $[a, b]$, and let $P = \{t_0, t_1, \ldots, t_n\}$ be a partition of $[a, b]$. To define a *Riemann sum* for f with respect to P, pick *any* points $c_i \in [t_{i-1}, t_i]$, one for each subinterval for P. The corresponding Riemann sum is

$$\begin{aligned} R(f, P) &:= f(c_1)(t_1 - t_0) + f(c_2)(t_2 - t_1) + \cdots + f(c_n)(t_n - t_{n-1}) \\ &= \sum_{i=1}^{n} f(c_i)(t_i - t_{i-1}). \end{aligned}$$

Since there are many choices for the c_i, there are many possible Riemann sums for the same partition. One nice thing about Riemann sums is that they don't involve computing least upper bounds or greatest lower bounds for heights. Another is that, as part (b) hints, they will usually give a better estimate of the integral than an upper or lower sum.

PROBLEM 1. Using the above notation, let

$$M_i = \operatorname{lub} f([t_{i-1}, t_i]) \quad \text{and} \quad m_i = \operatorname{glb} f([t_{i-1}, t_i]),$$

as usual. Your answers to the following should consist of complete sentences:

(a) Explain why $m_i \leq f(c_i) \leq M_i$ for $i = 1, \ldots n$.

(b) Explain why $L(f, P) \leq R(f, P) \leq U(f, P)$.

PROBLEM 2. Let $f(x) = x^2 + 1$ on the interval $[0, 3]$. Let $P = \{0, 1, 2, 3\}$ be a partition of $[0, 3]$.

(a) Compute a Riemann sum $R(f, P)$ by choosing c_1, c_2, and c_3 to be the midpoints of the three subintervals.

(b) Compute $L(f, P)$ and $U(f, P)$, and check that

$$L(f, P) \leq R(f, P) \leq U(f, P).$$

(c) Compute the exact area under f on this interval by using the fundamental theorem of calculus to compute $\int_0^3 f(x)\, dx$. (You will see that the Riemann sum you computed is a fairly good approximation.)

Problem 3. Suppose that $\int_0^3 g(x)\,dx = 4$, $\int_3^5 g(x)\,dx = 6$, and $\int_0^5 h(x)\,dx = -3$. Compute the following:

(a) $\int_0^5 g(x)\,dx$.

(b) $\int_0^5 (3\,g(x) + 5\,h(x))\,dx$.

Problem 4. You may find our essential derivatives handout helpful for this problem. (There is a link to it on our homepage.) Compute the following indefinite integrals (remember to add a constant c to each answer since the antiderivative is only determined up to the addition of a constant):

(a) $\displaystyle\int (x+3)\,dx$.

(b) $\displaystyle\int (x^{3/2} + 2x + 1)\,dx$.

(c) $\displaystyle\int \frac{1}{x^3}\,dx$.

(d) $\displaystyle\int (\smiley + 1)(2\,\smiley - 3)\,d\smiley$.

(e) $\displaystyle\int y^2\sqrt{y}\,dy$.

(f) $\displaystyle\int dx$.

(g) $\displaystyle\int (t^2 - \sin(t)\,dt$.

(h) $\displaystyle\int (\tan^2(\theta) + 1)\,d\theta$.

(i) $\displaystyle\int 7e^x\,dx$.

(j) $\displaystyle\int \frac{1}{x}\,dx$.