Math 111 Homework for Friday, Week 8

Problem 1. Consider the function

$$S(x) = x + \frac{1}{x}$$

for $x > 0$.

(a) Use the derivative of S to decide where S is decreasing and where S is increasing on $(0, \infty)$.

(b) Does S have a minimum on $(0, \infty)$? If so, what is the minimal value, and where does it occur? Justify your solution in any case. We have just seen that S is decreasing while $0 < x < 1$ and increasing when $x > 1$. Therefore, the minimum of S occurs at when $x = 1$. The minimum value is $S(1) = 1 + 1/1 = 2$.

Oops! I accidentally included the answer to question (b), which also indicates what happens in (a). Let's handle my mistake like this: First, turn in the solution to (a). Note that (a) requires a calculation of a derivative and then an argument that connects that calculation to the behavior of the function S. Second, please understand the solution to (b) that I provided, but you do not have to turn it in.

Problem 2. Find the point $(x, \sqrt{x})$ on the graph of $f(x) = \sqrt{x}$ that is closest to the point $(4, 0)$. **Justify your solution.** Recall that by the Pythagorean theorem, the distance between two points (a, b) and (c, d) in the plane is $d = \sqrt{(a-c)^2 + (b-d)^2}$. To make the calculation easier, note that the distance, d, is minimal exactly when d^2 is minimal. You can justify your solution by analyzing a derivative as in problem 1.

Problem 3. For each of the following sets X, answer the following questions:

(i) Is X bounded above? If so: (i) what is its least upper bound, $\text{lub}(X)$, and (ii) is $\text{lub}(X)$ an element of X?

(ii) Is X bounded below? If so: (i) what is its greatest lower bound, $\text{glb}(X)$, and (ii) is $\text{glb}(X)$ an element of X?

(a) $X = \{23, \pi, 1\}$, as set of three real numbers.

(b) $X = (-3, 10] = \{x \in \mathbb{R} : -3 < x \leq 10\}$, and interval.

(c) $X = \{-(1 + \frac{1}{1}), (1 + \frac{1}{2}), -(1 + \frac{1}{3}), (1 + \frac{1}{4}), -(1 + \frac{1}{5}), \dots\}$, an infinite sequence of real numbers.