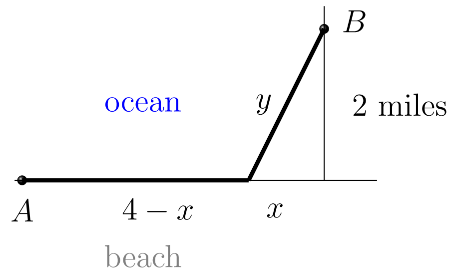
Math 111 Homework for Friday, Week 7

PROBLEM 1. You are working as a bartender in downtown Portland serving cocktails, one of which features a large spherical ice cube as pictured below:

It seems reasonable to assume that the ice is melting (i.e., its volume is decreasing) at a rate proportional to its surface area. As the sphere melts, its radius decreases, but does the radius decrease more or less quickly as time goes on? In other words, is the rate of change of the *radius* of the sphere increasing, decreasing, or remaining constant over time? These are the kinds of questions you are thinking about.

Use calculus to answer the question. You'll need the following: the volume of a sphere is $V = \frac{4}{3}\pi r^3$, and the surface area of a sphere is $S = 4\pi r^2$ where r is the radius. To say that x is *proportional* to y means there is a constant α such that $x = \alpha y$.

PROBLEM 2. A town is located on the coast at the point labeled A below. Four miles down the beach and 2 miles out is an island, labeled B below. We would like to connect A and B with a cable (darkened below). Part of the cable runs along the beach, and the rest goes across water. To install the cable in the ocean costs twice as much as installing it along the beach.



Thus, for our purposes, using the variables given in the picture, we can take the cost to be

$$C = (4-x) + 2y.$$

We would like to minimize the cost. This is an optimization problem, so let's apply our technique.

(a) Write C as a function of x.

(b) Identify the closed bounded interval to which x belongs for the purposes of our problem. (This allows us to invoke the extreme value theorem.)

(c) The cost, C, is a continuous function of x. So confined to the closed interval you just identified, according to the extreme value theorem, it will have a minimum and a maximum, and they will occur either at an endpoint of the interval or at a point at which the derivative of C is 0. To get started, compute the derivative of C with respect to x. (Recall that the derivative of $\sqrt{x}$ is $\dfrac{1}{2\sqrt{x}}$. You'll also need to use the chain rule.)

(d) Find any points x in your interval at which $C'(x) = 0$.

(e) Compute, using a calculator, the value of C at the endpoints of the interval and at the point where $C'(x) = 0$. Identify the minimum and the maximum values.