

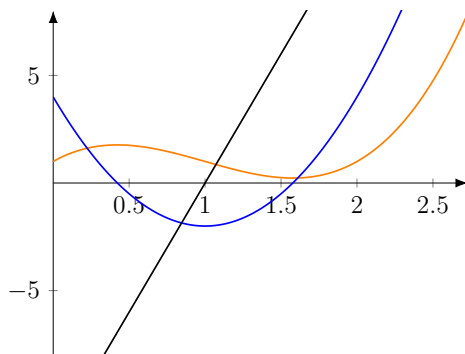
Math 111 Homework for Friday, Week 5

PROBLEM 1. Use the definition of the derivative,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h},$$

to compute the derivative of $f(x) = 1/x^2$. Show your work, as usual.

PROBLEM 2. The figure below contains the graphs of three functions: $f(x)$, its derivative $f'(x)$, and the derivative of $f'(x)$, denoted $f''(x)$. Which is which?



Graph of $f(x)$, $f'(x)$, and $f''(x)$.

PROBLEM 3. Let $f(x) = x^2 - 2x + 3$.

- (a) Compute $f'(x)$ using our derivative theorem in several steps as in the examples we did in class illustrating the use of the derivative theorem. You need to compute the derivative using just the following facts: (i) the derivative of a sum of two functions is the sum of the derivatives, (ii) the product rule for the derivative of the product of two function, (iii) the derivative of a constant is zero, and (iv) the derivative of $f(x) = x$ is 1. Show your steps.
- (b) (i) For which values of x is $f'(x) < 0$? What does that mean about the graph of f ?

- (ii) For which value of x is $f'(x) = 0$? What does that mean about the graph of f ?
 - (iii) For which values of x is $f'(x) > 0$? What does that mean about the graph of (x) ?
- (c) Use the information about the graph you've just gathered from the derivative to plot $f(x)$. **Label the points** where the graph hits the y -axis and where the derivative is 0 with their coordinates.

PROBLEM 4. Compute the derivatives of the following polynomials. You do not need to show your work—use this exercise as an excuse to learn how to take the derivative of a polynomial in your head.

- (a) $4x^3 - 2x^2 + 5x + 6$.
- (b) $x^5 + 3x^4 + x^2 + 3x + 1$.
- (c) $x^{16} + 4x^4 + 12$.

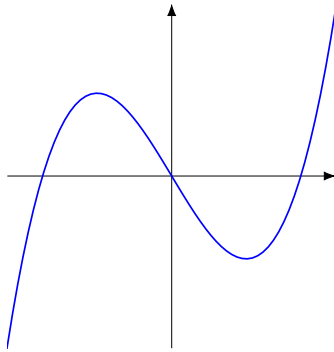
PROBLEM 5. Here are derivatives of some common functions:

$$(\ln(x))' = \frac{1}{x}, \quad (e^x)' = e^x, \quad (\sin(x))' = \cos(x), \quad (\cos(x))' = -\sin(x).$$

Use these facts and our derivative theorem (i.e., the sum, product, and quotient rules) to compute the derivatives of the following functions. Show your work.

- (a) $x \ln(x)$.
- (b) $\cos(x) + x^4 \sin(x)$.
- (c) $e^x \sin(x) + x^2$.

PROBLEM 6. The graph of a function f is show below. Draw this graph and overlay it with the graph of f' so that we can see the relation between the two.



Graph of a function.

PROBLEM 7. Compute the equation of the tangent line to $f(x) = \sin(x)$ at the point $x = 3\pi/4$. Draw the graph of $f(x)$ and this tangent line. Show your work.