Math 111 Homework for Friday, Week 4

PROBLEM 1. Define

$$g(x) = \begin{cases} x & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 3x - 2 & \text{if } x > 1. \end{cases}$$

(a) Graph g.

(b) Evaluate $\lim_{x\to 1} g(x)$ by computing both $\lim_{x\to 1^+} g(x)$ and $\lim_{x\to 1^-} g(x)$. (You do not need to justify your computation, but notice that on both sides of 1, the function g is linear—so the limits are easy. Recall that a limit exists at some point $x = c$ if and only if both the right- and left-hand limits exist at c and are equal.)

(c) Why isn't g continuous at 1?

(d) What part of the definition of the limit allows $\lim_{x\to 1} g(x)$ to exist even when g is not continuous at 1?

PROBLEM 2. Let $f(t) = 3t^2 - t + 2$, and think of f as describing the motion of a particle along the y-axis.

(a) What is the average speed of the particle between the times $t = 2$ and $t = 4$?

(b) Use the definition of the derivative to compute $f'(2)$. (Show your work in computing the required limit. Be careful not to drop the limit sign across equalities until the last step in your computation.)

(c) Find the equation for the tangent line for f at $t = 2$.

PROBLEM 3. Suppose f is a differentiable function whose derivative has the following properties:

- $f'(x) > 0$ for $x < 1$,

- $f'(x) = 0$ for $x = 1$,
- $f'(x) > 0$ for $1 < x < 2$,
- $f'(x) = 0$ for $x = 2$,
- $f'(x) > 0$ for $x > 2$.

Sketch a possible graph of f.

Problem 4. The derivative of $\cos(x)$ is $-\sin(x)$, i.e., $\cos'(x) = -\sin(x)$.

(a) Use the product rule to compute the derivative of $x^3\cos(x)$.

(b) Use the quotient rule to compute the derivative of $\dfrac{x^5}{\cos(x)}$.