Math 111 Homework for Friday, Week 2

Problem 1. Let $f(x) = \frac{\cos(x)-1}{x}$, defined for all real numbers $x \neq 0$. You will need a calculator of some sort for this problem. (Make sure x is in radians, not degrees.)

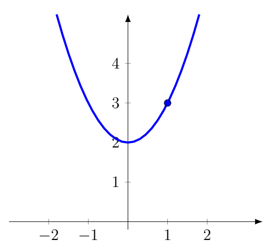
(a) Fill in the following table, rounding to four digits to the right of the decimal:

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

(b) Based on your table, what is your guess for the value of $\lim_{x\to 0} f(x)$?

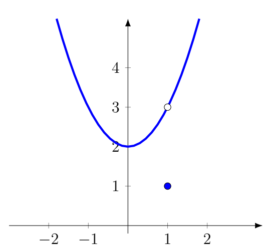
Problem 2. For the following, use the graph to either find the value of the limit or determine that the limit does not exist.

(a) $\lim_{x\to 1}(x^2+2)$.



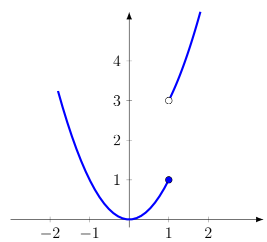
(b) $\lim_{x\to 1} f(x)$, where

$$f(x) = \begin{cases} x^2+2 & \text{if } x \neq 1 \\ 1 & \text{if } x = 1. \end{cases}$$



(c) $\lim_{x\to 1} g(x)$, where

$$g(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ x^2 + 2 & \text{if } x > 1. \end{cases}$$



PROBLEM 3. Give an ε-δ proof that $\lim_{x\to 3} 6x - 4 = 14$. Try to emulate the similar proofs given in class as closely as possible. Make sure that your proof consists of complete sentences! You could use Example 2.40 from the text as a template, too (although, I would use the word "Then" where they use "Thus"). Using their sentences and filling in the blanks is the proof. The solution that appears in the text includes a lot of scratch-work which you should not include in your proof.

PROBLEM 4. Let $f(x) = x^3$. Give an ε-δ proof that $\lim_{x\to 0} f(x) = 0$. Do not use the limit theorems given in class. (Hint: start by letting $\varepsilon > 0$. You now need to find $\delta > 0$ such that $0 < |x - 0| < \delta$ implies $|f(x) - 0| < \varepsilon$. In other words, you want $0 < |x| < \delta$ to imply $|x^3| < \varepsilon$. Which δ works? Once you've figured that out, try to emulate the write-up for the proof you gave in previous problem as closely as possible.)