

Math 111

December 5, 2022

Today

- ▶ Lotka-Volterra predator-prey model.

Lotka-Volterra predator-prey model

See the [Wikipedia page](#).

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The Lotka-Volterra equations modeling the evolution of these populations:

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Suppose $a = 2, b = 1, c = 0.25,$ and $d = 1$:

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1. When is the population of x growing? When is the population of y growing?
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2. What type of growth for $x'(t)$ is predicted if $y(t)$ is very small? What type of growth for $y'(t)$ is predicted if $x(t)$ is very small?
3. Under what condition is $x'(t) = 0$? Under what condition is $y'(t) = 0$. How about both $x'(t) = 0$ and $y'(t) = 0$?

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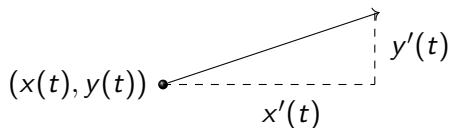
How can we keep track of $x(t)$, $y(t)$, $x'(t)$ and $y'(t)$ all at once?

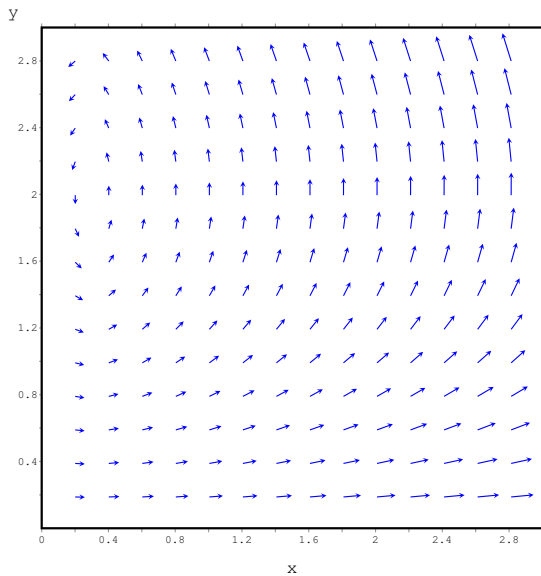
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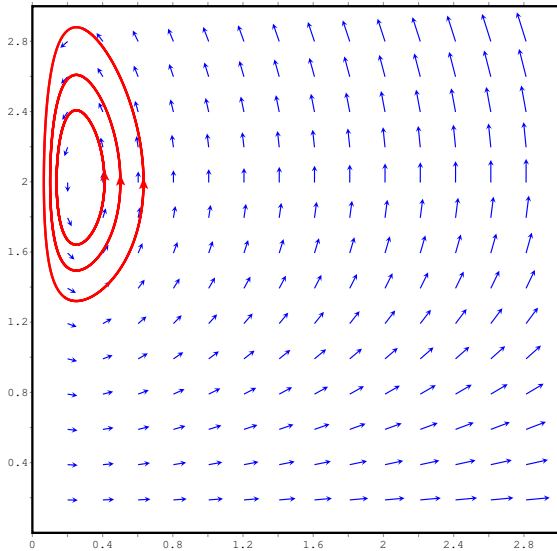
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Use vectors:





y



x