Math 111

November 11, 2022

Quiz on Monday

At the beginning of class on Monday, we will have a short quiz covering some subset of the following:

- The definition of the limit of a function.
- The definition of continuity.
- The definition of the derivative of a function.
- An example of computing the derivative of a particular function directly from the definition of the derivative.

For the first two items see the lecture notes for the classes on Friday, Week 1, and Wednesday, Week 3, and for the last two items, see the lecture notes for the class on Monday, Week 4.

See Monday, Week 11, on our class webpage for a copy of the above list.

Today

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Techniques of integration.

- Substitution.
- Integration by parts.

Substitution

Recall the chain rule: $(f(g(x)))' = f'(g(x))g'(x)$.

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Substitute back to get solution: $\frac{1}{7}(x^3+5)^7 + c$.

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Integrate $\int x^4 \cos(x^5)\,dx$.

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Write x in terms of u: $u = 1 + 5x \Rightarrow x = (u-1)/5$.

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$$x\sqrt{1+5x}\,dx = \frac{1}{5}\,x\sqrt{u}\,du = \frac{1}{25}(u-1)\sqrt{u}\,du.$$

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\end{aligned}$$

Substition: warning when evaluating definite integral

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$$\int_0^1 x^4(x^5+1)^6\,dx = \frac{1}{5}\int_1^2 u^6\,du = \frac{1}{35}u^7\Big|_1^2 = \frac{1}{35}(2^7 - 1^7) = \frac{127}{35}.$$

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Answer: Since $u = x^5 + 1$, it follows that $u = 1$ when $x = 0$,

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Why did the limits of integration change when we made the u-substitution?

Answer: Since $u = x^5 + 1$, it follows that $u = 1$ when $x = 0$, and $u = 2$ when $x = 1$.

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As an alternative, compute the indefinite integral using the substitution $u = x^5 + 1 = u$ but then substitute again at the end to get everything in terms of x:

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Then use the FTC:

$$\int_0^1 x^4(x^5+1)^6\,dx = \frac{1}{35}(x^5+1)^7\Big|_0^1 = \frac{127}{35}.$$

This way we did not have to change the limits of integration.

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We now modify the notation to specify the argument of the function (the independent variable):

$$u(x)v(x) = \int u'(x)v(x)\,dx + \int u(x)v'(x)\,dx.$$

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Using the notation $du = u'(x)dx$ and $dv = v'(x)dx$, we can write

$$uv = \int v\,du + \int u\,dv.$$

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Using the notation $du = u'(x)dx$ and $dv = v'(x)dx$, we can write

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Rearranging, we get the form that is useful for integration:

$$\boxed{\int u\,dv = uv - \int v\,du.}$$

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$$\boxed{\int u\,dv = uv - \int v\,du.}$$

Example. Compute $\int xe^x\,dx$.

Solution. We need to break up xe^x into a u part and a v part. There are two obvious choices here. The one that works is

$$\begin{aligned} u &= x \\ dv &= e^x\,dx \end{aligned}$$

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Solution. We need to break up xe^x into a u part and a v part. There are two obvious choices here. The one that works is

$$u = x$$
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We then need to find du and v:

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We then need to find du and v:

$$\begin{aligned} u &= x & \qquad du &= dx \\ dv &= e^x\,dx & v &= e^x. \end{aligned}$$

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Solution. Continuing,

$$\begin{aligned} u &= x & du &= dx \\ dv &= e^x\,dx & v &= e^x. \end{aligned}$$

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Example. Compute $\int xe^x\,dx$.

Solution. Continuing,

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Using the integration by parts formula:

$$\int xe^x\,dx = \int u\,dv$$

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Example. Compute $\int xe^x\,dx$.

Solution. Continuing,

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Using the integration by parts formula:

$$\begin{aligned} \int xe^x\,dx &= \int u\,dv \\ &= uv - \int v\,du \\ &= xe^x - \int e^x\,dx \\ &= xe^x - e^x + c. \end{aligned}$$

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Check that $\int xe^x \, dx = xe^x - e^x + c$:

$$\begin{aligned} (xe^x - e^x + c)' &= (xe^x)' - (e^x)' + (c)' \\ &= (e^x + xe^x) \end{aligned}$$

Integration by parts

Check that $\int xe^x \, dx = xe^x - e^x + c$:

$$\begin{aligned}(xe^x - e^x + c)' &= (xe^x)' - (e^x)' + (c)' \\ &= (e^x + xe^x) - e^x\end{aligned}$$

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Thus, for example,

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$$\int_0^1 xe^x\,dx$$

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Thus, for example,

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(xe^x - e^x + c)' &= (xe^x)' - (e^x)' + (c)' \\
&= (e^x + xe^x) - e^x + 0 \\
&= xe^x.
\end{aligned}
$$

Thus, for example,

$$
\begin{aligned}
\int_0^1 xe^x \, dx &= (xe^x - e^x)\Big|_0^1 \\
&= (1 \cdot e^1 - e^1) - (0 \cdot e^0 - e^0)
\end{aligned}
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Thus, for example,

$$\begin{aligned} \int_0^1 xe^x \, dx &= (xe^x - e^x)\Big|_0^1 \\ &= (1 \cdot e^1 - e^1) - (0 \cdot e^0 - e^0) \\ &= (e - e) - (0 - 1) \end{aligned}$$

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$$\begin{aligned}(xe^x - e^x + c)' &= (xe^x)' - (e^x)' + (c)' \\ &= (e^x + xe^x) - e^x + 0 \\ &= xe^x.\end{aligned}$$

Thus, for example,

$$\begin{aligned}\int_0^1 xe^x\,dx &= \left(xe^x - e^x\right)\Big|_0^1 \\ &= (1\cdot e^1 - e^1) - (0\cdot e^0 - e^0) \\ &= (e - e) - (0 - 1) \\ &= 1.\end{aligned}$$

(Sage demo.)

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Solution. Integrate by parts:

$$\begin{aligned} u &= x & du &= dx \\ dv &= \cos(x)\, dx & v &= \sin(x). \end{aligned}$$

Integration by parts

Example. Compute $\int x \cos(x)\, dx$.

Solution. Integrate by parts:

$$\begin{aligned} u &= x & du &= dx \\ dv &= \cos(x)\, dx & v &= \sin(x). \end{aligned}$$

Then

$$\int x \cos(x)\, dx = \int u\, dv$$

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Then

$$\begin{aligned} \int x\cos(x)\,dx &= \int u\,dv \\ &= uv - \int v\,du \\ &= x\sin(x) - \int \sin(x)\,dx \\ &= x\sin(x) + \cos(x) + c. \end{aligned}$$

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$$
\begin{aligned}
(x\sin(x) + \cos(x) + c)' &= (x\sin(x))' + \cos'(x) \\
&= (\sin(x) + x\cos(x)) - \sin(x) \\
&= x\cos(x).
\end{aligned}
$$

Nice puzzle. Compute $\int e^x \cos(x)\,dx$ by parts.