Math 111

September 30, 2022

Goals

- Chain rule practice.
- Method for related rates problems.

Chain rule review

Suppose x and y are functions of t. What is the derivative of the following function with respect to t?

$$x^3 + y^5$$

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Solution. We have

$$\frac{d}{dt}(x^3 + y^5) = 3x^2\frac{dx}{dt} + 5y^4\frac{dy}{dt}.$$

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&= 4x^3 \frac{dx}{d\text{👻}} \cdot \sin(y^2) + x^4 \cos(y^2)(2y) \frac{dy}{d\text{👻}}
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&= 4x^3 \sin(y^2) \frac{dx}{d\mathghost} + 2x^4 y \cos(y^2) \frac{dy}{d\mathghost}.
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&= e^{x^3y^4}\left(3x^2y^4\frac{dx}{dt} + 4x^3y^3\frac{dy}{dt}\right).
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Related rates

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Continuing, we see that as long as $t \neq 0$, we can cancel it to get

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$$\frac{dz}{dt} = \frac{60^2 + 50^2}{\sqrt{60^2 + 50^2}} = \sqrt{60^2 + 50^2} \quad \text{provided } t \neq 0.$$

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So as we decrease the volume, the pressure will increase. Suppose we decrease the volume at a constant rate. Does the pressure increase at a constant rate?