Math 111

September 21, 2022

Goals

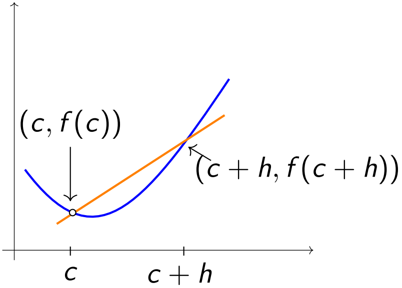
- Practice with the derivative:
  - Use the definition to compute it in specific cases.
  - Find the equation of the tangent line.
- Introduce a derivative theorem analogous to the limit theorem we saw previously.

Announce tomorrow's math talk.

Review

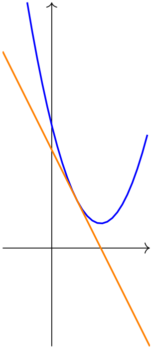
Average rate of change: $\dfrac{f(c+h)-f(c)}{h}$

Instantaneous rate of change: $f'(c) = \lim_{h \to 0} \dfrac{f(c+h)-f(c)}{h}$



Review

$$\begin{aligned} f'(c) &= \text{derivative at } c \\ &= \text{instantaneous rate of change of } f \text{ at } c \\ &= \text{slope of } f \text{ at } c \end{aligned}$$



Practice

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SOLUTION:

$$\text{average speed} = \frac{f(4) - f(1)}{4 - 1}$$

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$$= \lim_{h \to 0} \frac{\sqrt{1+h} - 1}{h}$$

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SOLUTION:

$$
\begin{aligned}
f'(1) &= \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{1+h} - 1}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{1+h} - 1}{h} \cdot \frac{\sqrt{1+h} + 1}{\sqrt{1+h} + 1}
\end{aligned}
$$

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$$= \lim_{h \to 0} \frac{(1+h) - 1}{h(\sqrt{1+h} + 1)}$$

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$$1 = \frac{1}{2} \cdot 1 + b,$$

which implies $b = \frac{1}{2}$.

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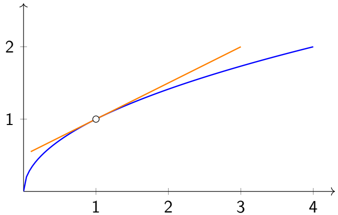
$$1 = \frac{1}{2} \cdot 1 + b,$$

which implies $b = \frac{1}{2}$. So the tangent line has equation

$$\frac{1}{2}x + \frac{1}{2}.$$



Graph of $f(x) = \sqrt{x}$ and its tangent line at $x = 1$.

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Review

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So we have shown

$$\left(x^{\frac{1}{2}}\right)' = \frac{1}{2}x^{-\frac{1}{2}}.$$

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So we have shown

$$\left(x^{\frac{1}{2}}\right)' = \frac{1}{2} x^{-\frac{1}{2}}.$$

Compare this to the rule we saw last time: $(x^n)' = nx^{n-1}$ for $n = 1, 2, 3, \ldots$

Derivative theorem

Theorem. Suppose f and g are differentiable functions at a point x.

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4. The *product rule or Leibniz rule*:
$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x).$$
5. The *quotient rule.*
$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}.$$

Derivative theorem

Shorthand:

- $(c)' = 0$ for all constants c
- $(x)' = 1$
- sum rule for derivatives: $(f+g)' = f' + g'$
- product rule for derivatives: $(fg)' = f'g + fg'$
- quotient rule for derivatives:

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Consequences of our derivative theorem

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Proof. Apply the product rule and the fact the derivative of a constant is 0:

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Let c be a real number, and let f be a differentiable function. Then

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Proof. Apply the product rule and the fact the derivative of a constant is 0:

$$(cf)' = (c)'f + cf' = 0 \cdot f + cf'$$

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Let c be a real number, and let f be a differentiable function. Then

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Proof. Apply the product rule and the fact the derivative of a constant is 0:

$$(cf)' = (c)'f + cf' = 0 \cdot f + cf' = cf'$$

Example. Since $(x^5)' = 5x^4$,

$$(7x^5)' = 7(x^5)' = 7 \cdot 5x^4 = 35x^4.$$

Consequences of our derivative theorem

$$(f - g)' =$$

Consequences of our derivative theorem

$$(f - g)' = (f + (-g))'$$

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$$(x^2)' = (x \cdot x)'$$

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$$(x^2)' = (x \cdot x)' = (x)'x + x(x)'$$

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$$(x^2)' = (x \cdot x)' = (x)'x + x(x)' = 1 \cdot x + x \cdot 1 = 2x$$

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(x^3)' &= (x \cdot x^2)' = (x)'x^2 + x(x^2)' \\
&= 1 \cdot x^2 + x(2x) = 3x^2
\end{aligned}
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$$(x^4)' = (x \cdot x^3)' = (x)'x^3 + x(x^3)'$$

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(x^4)' &= (x \cdot x^3)' = (x)'x^3 + x(x^3)' \\
&= 1 \cdot x^3 + x(3x^2) = 4x^3
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etc.

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$$(3x^5 - 2x^2 - 7)'$$

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$$(3x^5 - 2x^2 - 7)' = (3x^5)' + (-2x^2)' + (-7)'$$

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$$
\begin{aligned}
(3x^5 - 2x^2 - 7)' &= (3x^5)' + (-2x^2)' + (-7)' \\
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$$

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\begin{aligned}
(3x^5 - 2x^2 - 7)' &= (3x^5)' + (-2x^2)' + (-7)' \\
&= 3(x^5)' - 2(x^2)' + 0 \\
&= 3(5x^4) - 2(2x)
\end{aligned}
$$

Consequences of our derivative theorem

$$\begin{aligned}(3x^5 - 2x^2 - 7)' &= (3x^5)' + (-2x^2)' + (-7)' \\ &= 3(x^5)' - 2(x^2)' + 0 \\ &= 3(5x^4) - 2(2x) \\ &= 15x^4 - 4x.\end{aligned}$$

Consequences of our derivative theorem

$$(4x^3 - 9x^2 + 3x + 4)'$$

Consequences of our derivative theorem

$$(4x^3 - 9x^2 + 3x + 4)' = (4x^3)' + (-9x^2)' + (3x)' + (4)'$$

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$$\begin{aligned}(4x^3 - 9x^2 + 3x + 4)' &= (4x^3)' + (-9x^2)' + (3x)' + (4)' \\ &= 4(x^3)' + (-9)(x^2)' + 3(x)' + (4)'\end{aligned}$$

Consequences of our derivative theorem

$$\begin{aligned}(4x^3 - 9x^2 + 3x + 4)' &= (4x^3)' + (-9x^2)' + (3x)' + (4)' \\ &= 4(x^3)' + (-9)(x^2)' + 3(x)' + (4)' \\ &= 4(3x^2) - 9(2x) + 3 \cdot 1 + 0\end{aligned}$$

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Consequences of our derivative theorem

$$(-6x^5 + 4\sqrt{x})'$$

Consequences of our derivative theorem

$$(-6x^5 + 4\sqrt{x})' = (-6x^5)' + (4\sqrt{x})'$$

Consequences of our derivative theorem

$$
\begin{aligned}
(-6x^5 + 4\sqrt{x})' &= (-6x^5)' + (4\sqrt{x})' \\
&= (-6)(x^5)' + 4(\sqrt{x})'
\end{aligned}
$$

Consequences of our derivative theorem

$$\begin{aligned}
(-6x^5 + 4\sqrt{x})' &= (-6x^5)' + (4\sqrt{x})' \\
&= (-6)(x^5)' + 4(\sqrt{x})' \\
&= (-6)(5x^4) + 4\left(\frac{1}{2\sqrt{x}}\right)
\end{aligned}$$

Consequences of our derivative theorem

$$\begin{aligned}
(-6x^5 + 4\sqrt{x})' &= (-6x^5)' + (4\sqrt{x})' \\
&= (-6)(x^5)' + 4(\sqrt{x})' \\
&= (-6)(5x^4) + 4\left(\frac{1}{2\sqrt{x}}\right) \\
&= -30x^4 + \frac{2}{\sqrt{x}} \\
&= -30x^4 + 2x^{-1/2}.
\end{aligned}$$

HW problem

PROBLEM. Suppose that $\lim_{x \to 0} f(x) = 1$. Use the definition of the limit with $\varepsilon = 1$ to show that there must be some open interval about 0 such that $f(x) > 0$ for every x in that interval, except possibly at $x = 0$.