Math 111

August 29, 2022

Today

- Course organization.
- To do list for Wednesday's class.
- Overview of Calculus.

Course organization and To Do list for Wednesday

See our course homepage:

https://people.reed.edu/~davidp/111/.

Overview of Calculus

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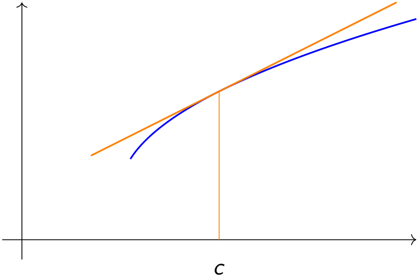
It applies this idea to two seemingly unrelated topics: *rates of change* and *areas*.

I. Derivatives: rates of change



Graph of a function g and its best linear approximation at the point c.

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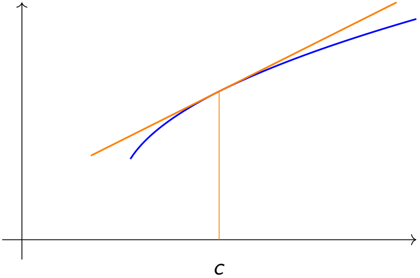

Graph of a function g and its best linear approximation at the point c.

The slope of the line gives the rate of change of the function at the point c.

Rate of change for time-distance graph = velocity

Imagine a particle moving along the real number line, and let $g(t)$ be the distance of the particle from the origin at time t.

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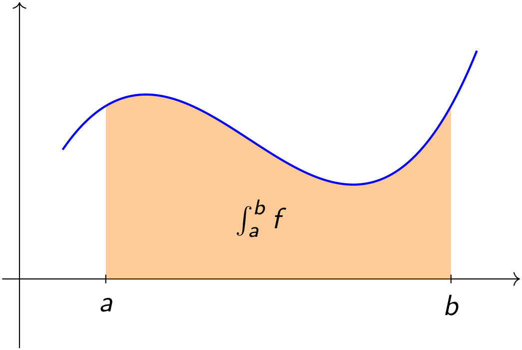
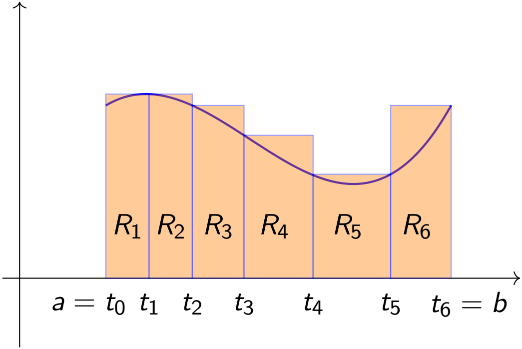
Desmos demonstration

Integrals: area under graph



The integral $\int_a^b f$ is the area under the graph of f from $t = a$ to $t = b$.

Approximate the area with rectangles



Approximating the area under the graph of f with rectangles.

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The integral of the derivative gives the net change.

Summary of goals for Math 111

- What is speed? (derivatives)
- What is area? (integrals)
- How are they related? (Fundamental Theorem of Calculus (FTC))
- Theory:
  - IVT (intermediate value theorem)
  - EVT (extreme value theorem)
  - MVT (mean value theorem)
  - Chain rule, product rule
  - FTC.
- Applications:
  - Calculate speed and area efficiently.
  - Optimization (maximize and minimize functions).
  - Related rates.
  - Differential equations and population models.

Deep technical definition

Definition. Let f be a function defined in an open interval containing a point c, except f might not be defined at the point c, itself. Let L be a real number. The *limit of $f(x)$ as x approaches c* is L, denoted $\lim_{x \to c} f(x) = L$, if for all $\varepsilon > 0$, there exists $\delta > 0$ such that

$$0 < |x - c| < \delta$$

implies

$$|f(x) - L| < \varepsilon.$$