Joint Quantum Measurement Using Unbalanced Array Detection

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We have measured the joint Q-function of a highly multimode field using unbalanced heterodyne detection with a charge-coupled device array detector. We use spectral interferometry between a weak signal field and a strong, 100 fs duration local oscillator pulse to reconstruct the joint quadrature amplitude statistics of about 25 temporal modes. By adjusting the time delay between the signal and local oscillator pulses we are able to shift all the classical noise to modes distinct from the signal. This obviates the need to use a balanced detector.

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A complete description of the quantum state of a multimode field requires simultaneous measurement of the joint statistics of all the modes. Joint statistics yield information about the correlations between modes that cannot be obtained from single-mode measurements. In this Letter we describe a new approach to measuring the joint statistics of a highly multimode field using a single array detector in an unbalanced configuration. Nonetheless, we are still able to eliminate the classical noise of the local oscillator (LO) by judicious placement of its modes with respect to those of the signal field.

To date joint measurement of two-mode fields has yielded determinations of the second-order coherence $g^{(2)}(t, t + \tau)$ [1], joint photon statistics [2], and the joint polarization state of the two-photon subspace of an entangled pair of photons [3]. The amount of data required and the use of balanced detection with pairs of photodiodes make it difficult to extend the techniques used in these experiments to fields with many modes. No measurements of the full joint quantum state of even a two-mode system have been reported.

Measurements of more than two modes at once can be made using array detection [4]. It has been demonstrated that it is possible to obtain simultaneous (but not joint) measurements of the Wigner functions of many modes by using array detectors [5,6]. It has been shown theoretically that one may use array detectors to measure the joint Q-function of a multimode field [7]. One of the difficulties of these methods is the need to balance two array detectors and perform pixel-by-pixel subtraction to eliminate the classical intensity noise of the LO [8]. However, we demonstrate here that it is possible to achieve the same effect by using only a single array, without balancing, by suitable arrangement of the LO and signal fields. Furthermore, we simultaneously measure many temporal modes without having to scan the time delay or the phase of the LO; this drastically reduces the amount of data that needs to be acquired. This measurement technique could be used, for example, to simultaneously determine two-time field and intensity correlations of a signal field containing only a few photons.

The technique of Fourier transform spectral interferometry allows us to achieve the shot-noise limit (SNL) in unbalanced detection by encoding the relevant information about the signal pulse using a carrier modulation in the spectral domain [9,10]. By measuring the spectrum $\tilde{S}(\omega)$ of an electric field which is given by the sum of an LO field $\tilde{E}_{\rm LO}$ and a signal field $\tilde{E}_S$, which is delayed from the LO field by a time τ, one obtains

$$\begin{aligned}\tilde{S}(\omega) &= |\tilde{E}_{\rm LO}(\omega) + \tilde{E}_S(\omega)\exp(i\omega\tau)|^2 \\ &= |\tilde{E}_{\rm LO}(\omega)|^2 + |\tilde{E}_S(\omega)|^2 \\ &\quad + [\tilde{E}^*_{\rm LO}(\omega)\tilde{E}_S(\omega)\exp(i\omega\tau) + \text{c.c.}]. \quad (1)\end{aligned}$$

Fourier transforming this measurement into the temporal domain yields

$$\begin{aligned}S(t) &= F\{\tilde{S}(\omega)\} \\ &= E^*_{\rm LO}(-t) \otimes E_{\rm LO}(t) + E^*_S(-t) \otimes E_S(t) \\ &\quad + f(t-\tau) + f^*(-t-\tau), \quad (2)\end{aligned}$$

where $f(t) = E^*_{\rm LO}(-t) \otimes E_S(t)$, and $\otimes$ denotes a convolution. The first term peaks at $t = 0$ and is the dominant term. It contains the second-order classical LO noise that would be removed if one were performing balanced detection using two arrays. If τ is large enough, $f(t-\tau)$, which peaks near $t = \tau$ and contains all the information on the heterodyned signal pulse, is temporally separated from the classical noise at $t = 0$. Thus, by temporally delaying the signal with respect to the LO and performing unbalanced detection we can eliminate the classical noise on the LO just as effectively as if we had performed balanced detection, without the experimental complication of accurately aligning the two arrays.

In our experiment the mode statistics are expressed in terms of the joint Q-function of the modes. The Q function is a quantum mechanical, phase-space, quasiprobability distribution; it is positive definite and may be used to calculate quantum expectation values of antinormally ordered operators [11,12]. In principle the Q function contains all information about the quantum mechanical state of a system. However, to extract the density matrix from the Q function it is necessary to perform a numerical deconvolution, which is impractical with real experimental data. A full quantum mechanical analysis is presented in Ref. [7]. Here we present a somewhat different analysis of this experiment, concentrating on the differences between the proposed experiment of Ref. [7] and our actual implementation based on unbalanced homodyne detection using a multimode LO pulse.

Following Ref. [7], we consider an optical spectrum measured by an array detector, with $\hat{n}_j = \hat{a}_j^\dagger \hat{a}_j$ being the operator corresponding to the number of photons incident on pixel j of the array. Each pixel measures a different spectral mode and the mode frequencies are given by $\omega_j = \overline{\omega} + j\delta\omega$, where $\overline{\omega}$ is the mean frequency of the field, $\delta\omega$ is the frequency separation of adjacent pixels, and there are N pixels labeled by $-N/2 \le j < N/2$. We can also express the measured field in terms of temporal modes $\hat{b}_k$, $-N/2 \le k < N/2$ using the Fourier relation

$$\hat{a}_j = \frac{1}{\sqrt{N}} \sum_k \exp[i2\pi jk/N]\hat{b}_k \,. \tag{3}$$

In our experiment the signal and LO fields are separated by a time delay. For the purposes of this analysis we thus assume that the LO occupies the $2M + 1$ temporal modes near the center of our time window. The signal occupies the temporal modes after the LO, and the modes before the LO are empty. In order to make this distinction between these modes clearer, we rewrite the mode operators as

$$\hat{b}_k = \begin{cases} \hat{b}_k^{(\mathrm{vac})} & -N/2 \le k < -M\,, \\ \hat{b}_k^{(\mathrm{lo})} & -M \le k \le M\,, \\ \hat{b}_k^{(s)} & M < k < N/2\,, \end{cases} \tag{4}$$

where the superscripts refer to vacuum, LO, or signal mode operators.

The operator that we measure corresponds to the Fourier transform of $\hat{n}_j$,

$$\hat{K}_p \equiv \sum_j \exp[-i2\pi pj/N]\hat{n}_j \,. \tag{5}$$

Equations (3) and (4) can be combined to express the spectral mode number operator $\hat{n}_j$ in terms of temporal mode operators $\hat{b}_k$. Terms of second order in operators corresponding to the weak fields $\hat{b}_p^{(s)}$ and $\hat{b}_p^{(\mathrm{vac})}$ are discarded. Substituting this expression for $\hat{n}_j$ into Eq. (5) yields an expression for $\hat{K}_p$ in terms of the $\hat{b}_k$'s. Furthermore, the modes of our LO pulse are in large amplitude coherent states, so the dominant contributions are retained if we replace the LO mode operators $\hat{b}_k^{(\mathrm{lo})}$ by their corresponding coherent state amplitudes β_k. The terms that contribute to the summations in the expression for $\hat{K}_p$ depend on the value of p; for $p > 2M$ we find

$$\hat{K}_p = \sum_{k=-M}^{M} (\beta_k^* \hat{b}_{k+p}^{(s)} + \beta_k \hat{b}_{k-p}^{\dagger(\mathrm{vac})}) \,. \tag{6}$$

If the LO occupies only a single ($k = 0$) temporal mode, then Eq. (6) simplifies to

$$\hat{K}_p = \beta_0^* \hat{b}_p^{(s)} + \beta_0 \hat{b}_{-p}^{\dagger(\mathrm{vac})} . \tag{7}$$

In this case a measurement of $\hat{K}_p$ returns a complex number, which from Eq. (7) we can interpret as a measurement of the signal mode annihilation operator $\hat{b}_p^{(s)}$, plus an added vacuum noise contribution. Since the annihilation operator is the sum of the two field quadrature amplitudes $\hat{b}_p^{(s)} = (1/2^{1/2})(\hat{x}_p + i\hat{y}_p)$, the real and imaginary parts of our measurement correspond to simultaneous measurement of the quadrature amplitudes x_p and y_p. The price we pay for simultaneous measurement of noncommuting observables is the presence of the additional vacuum noise, as was first pointed out by Arthurs and Kelly [12,13].

Each run of the apparatus returns a set of $(N/2) - M$ complex numbers, which correspond to the quadrature amplitudes of the signal modes (x_p, y_p). By histogramming our measured values of x_p and y_p we create a probability distribution, which in the limit of a large number of samples tends to the Q distribution for the field quadratures $Q(x_p, y_p)$ [11,12,14]. Indeed, since we simultaneously measure the quadrature amplitudes for all values of p, we can create joint Q-distributions for multiple modes. It is difficult to display these multivariable distributions graphically, so here we display correlations between the modes in terms of the joint distribution of the x quadratures of the different modes. These distributions are given by the marginals

$$Q'(x_p, x_{p'}) = \iint Q(x_p, y_p, x_{p'}, y_{p'})\, dy_p\, dy_{p'} \,. \tag{8}$$

In our experiments, the LO pulse does not occupy a single temporal mode, so our measurements are more accurately described by Eq. (6) than they are by Eq. (7). Thus, the quadrature amplitudes for the measured mode do not represent those of a specific temporal mode but correspond to a linear combination of modes. As seen from Eq. (6), the mode we actually measure represents a convolution between the LO and signal modes. Despite this fact, it is straightforward to demonstrate that $[\hat{K}_p, \hat{K}_{p'}] = 0$ for $p, p' > 2M$, so the modes we measure are independent. It is possible to deconvolve the known LO field from the data to obtain higher temporal resolution, provided the LO spectrum is broader than that of the signal.

The experimental apparatus is shown in Fig. 1. Well characterized 100 fs duration pulses from a mode-locked Ti:sapphire laser are frequency doubled to a wavelength of 400 nm [15]. The pulses pass through a 25 mm path length of lithium triborate (LBO) and a polarizer. The input polarization to the LBO is approximately 5° off of perfect alignment with one of the eigenpolarizations of the crystal. Since the two eigenpolarizations have different group velocities, the input pulse splits into two orthogonally polarized pulses that travel at different speeds. On output a second polarizer projects these two pulses onto the same

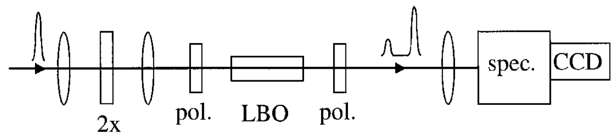

FIG. 1. The experimental apparatus. Here, "pol." stands for polarizer, 2x stands for frequency doubling crystal, and "spec." stands for spectrometer.

polarization axis. The polarizer angle can be varied in order to adjust the relative intensity of the two pulses, producing a weak signal pulse and a strong LO pulse that are separated in time by approximately 3 ps. This method of generating two pulses yields a very stable phase relationship between them. After the polarizer, we focus the beam onto the entrance slit of a 0.30 m spectrometer, and we detect the spectrum with a charge-coupled-device (CCD) array.

The CCD is a 512 $\times$ 512 array of pixels. We use an exposure time of 300 ms, so each "shot" is actually composed of millions of pulses, since our laser has a repetition rate of 82 MHz. The spectrum is focused onto approximately 15 rows, so we collect data from these rows, and sum along each vertical column to obtain a 512 point spectrum. The wavelength increment between adjacent pixels is 0.035 nm, and our resolution is about twice this. When we Fourier transform our spectra to obtain K_p the index p corresponds to a time index, where $t = p(29.8\text{ fs})$. Before we acquire data with a signal present we perform two background measurements. We first block all light impinging on the array to obtain a background dark level that we subtract. We then block the signal field and allow only the LO to pass; this creates a signal field in the vacuum. We acquire 100 shots with a vacuum signal and average these, which yields $\langle K_p \rangle_{\text{vac}}$, where the subscript indicates that the signal field is in the vacuum.

We acquire 5000–15 000 spectra with the signal present and Fourier transform them, on each shot obtaining values for K_p. From this we subtract the vacuum signal level to obtain $K_p - \langle K_p \rangle_{\text{vac}}$. We scale our measurements by the total number of detected photoelectrons n_t. After scaling we obtain the quadrature amplitudes

$$x_p = \frac{\sqrt{2}}{n_t}\,\text{Re}(K_p - \langle K_p \rangle_{\text{vac}}), \qquad y_p = \frac{\sqrt{2}}{n_t}\,\text{Im}(K_p - \langle K_p \rangle_{\text{vac}}), \tag{9}$$

which we histogram into a 32 $\times$ 32 array of bins to obtain the Q function. Furthermore, since the Q function allows the determination of antinormally ordered operators, we can directly calculate the mean $\langle \hat{n}_p^{(s)} \rangle$ and standard deviation $\langle (\Delta \hat{n}_p^{(s)})^2 \rangle^{1/2}$ of the number of photons in the pth signal mode by placing the appropriate operator in antinormal order.

In Fig. 2(a) we show the measured Q function of one mode of the signal field. This mode has a mean of 6.5 photons, and the fluctuations on the photon number are shot-noise limited with a standard deviation of 2.5. We have obtained SNL operation despite the fact that our LO beam has classical energy fluctuations and we have not used balanced detection. For this particular data set we acquired 5000 shots of data, and the standard deviation of the LO fluctuations was 2.6% of the mean, while the peak-to-peak fluctuation was over 15%.

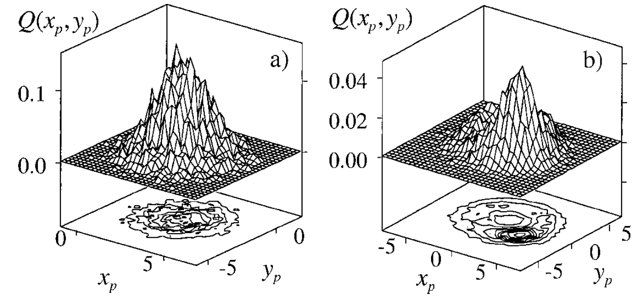

FIG. 2. The measured Q functions of a temporal mode for (a) a signal pulse with a stable phase and (b) a chirped signal pulse with a random phase.

When using the experimental arrangement in Fig. 1, the signal and LO pulses have the same duration, so we do not have very good time resolution of the signal pulse. Also, since the signal pulse is in a coherent state the measured Q functions and two-mode correlations are all Gaussian functions, which does not fully illustrate the power of our measurement technique to measure arbitrary states. To rectify this we have replaced the LBO and polarizers by a Michelson interferometer. The two arms of the interferometer form the signal and local oscillator beams, and we make the length of the signal arm longer in order to delay the signal pulse by $\sim$3.5 ps. The signal arm also contains 1.5 cm of glass that adds dispersion to the signal pulse; the signal pulse is thus stretched and chirped. The path length difference between the two arms is not stabilized; this randomizes the signal phase producing non-Gaussian Q functions.

Figure 2(b) shows the measured Q function for a mode in the trailing edge of the pulse. The Q function is circular;

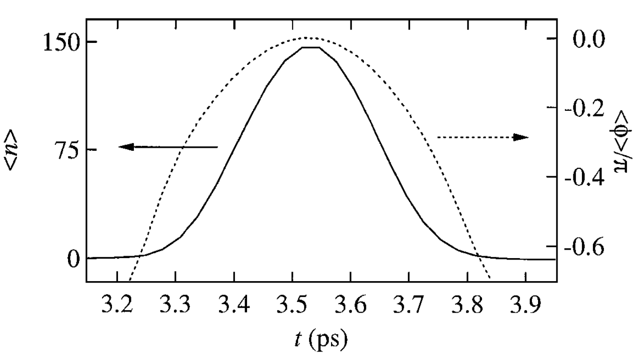

FIG. 3. The mean photon number $\langle n \rangle$ (solid curve) and phase $\langle \phi \rangle$ (dashed curve) of a chirped signal pulse as a function of delay time after the local oscillator pulse.

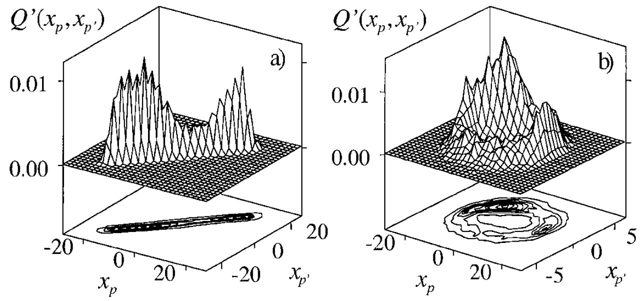

FIG. 4. The measured joint distributions $Q'(x_p, x_{p'})$ for two different pairs of temporal modes of a chirped signal pulse. The mode delay times in (a) are $t = 3.457$ ps and $t' = 3.576$ ps, while in (b) they are $t = 3.457$ ps and $t' = 3.785$ ps

however, there is still a peak in the distribution indicating that the phase has not been completely randomized. The interferometer phase drifted considerably at the beginning of the 8 h experiment but gradually stabilized toward the end of this 14 000 shot data run—if we analyze only the first 10 000 data points we obtain a Q function that is more nearly circular, while the last 4000 points yield a "banana-shaped" Q function. The number of photons in the mode corresponding to Fig. 2(b) is 5.9 ± 2.6, which is within 10% of the SNL. In Fig. 3 we show the mean photon number $\langle n \rangle$ and phase $\langle \phi \rangle = \langle \tan^{-1}[(y_p)/(x_p)] - \tan^{-1}[(y_{p'})/(x_{p'})]\rangle$ (where p' corresponds to a mode near the peak of the pulse) of our measured signal as functions of time. This shows that dispersion generates a phase across the pulse that is quadratic in time.

Figure 4 shows correlations between the x quadratures of two modes in terms of joint distributions $Q'(x_p, x_{p'})$. Figure 4(a) shows correlations between two modes that are both near the peak of the pulse and consequently have nearly the same amplitude and phase. This joint distribution lies mainly along a line whose slope is 1, indicating strong positive correlations between the x quadratures. This is what we would expect for two modes whose phases are nearly the same. Because the relative phase of the two modes is nearly constant but the absolute phase of each is largely random, we observe a pattern that is essentially a Lissajous figure representing the relative amplitude and phase of the two modes. In Fig. 4(b) we show correlations between a mode near the center of the pulse and another in the trailing edge. The average phase difference between these modes is nearly $\pi/2$, and this produces a joint distribution that is very different. While the joint distribution appears circular in Fig. 4(b), we note that the scales along the x_p and x_p' axes are quite different, so this distribution is actually elliptical. Again, such a joint distribution (Lissajous figure) is expected for two modes with random phases, but whose average phase difference is $\pi/2$.

We have demonstrated a new technique for performing joint quantum measurements of multitemporal mode optical fields, which results in the determination of the joint Q-function of the modes. This technique is based on unbalanced heterodyne detection with array detectors. The measurements are performed at the shot-noise limit, can detect modes containing only a few photons, and have high temporal resolution. We have shown that by using array detection we are able to effectively eliminate the classical noise on the local oscillator field without the need to perform balanced detection, which is a considerable experimental simplification.

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