

Math 387

Homework 7

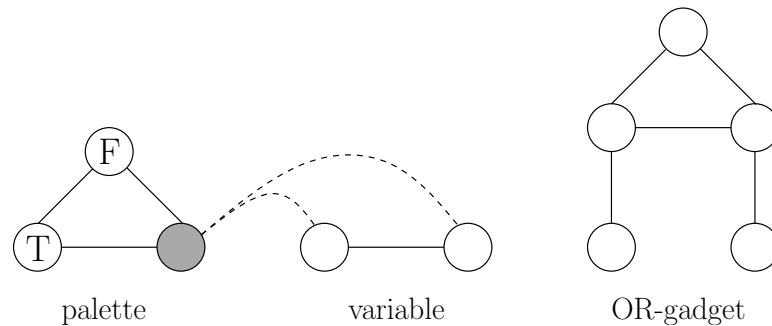
Due Friday, October 30

Practice exercises from the book

7.22, 7.25, 7.31, 7.34, 7.38

Problems

1. Take $DOUBLESAT = \{ \langle \phi \rangle \mid \phi \text{ is a boolean formula with at least two different satisfying assignments} \}$. Show that this language is NP-complete.
2. A *coloring* of a graph is an assignment of way of assigning a color to each vertex so that no two adjacent vertices have the same color. Take $3COLOR = \{ \langle G \rangle \mid G \text{ has a coloring that uses only three colors} \}$. Show that $3COLOR$ is NP-complete. (Hint: Use the following subgraphs.)



3. Let $SET-SPLITTING = \{ \langle S, C \rangle \mid S \text{ is a finite set and } C = \{C_1, \dots, C_k\} \text{ is a collection of subsets of } S \text{ such that each element of } S \text{ can be colored red or blue so that each } C_i \text{ has at least one element of each color} \}$. Show that this language is NP-complete.
4. Considering the following problem. You have a list of final exams and a list of students. Each student is taking some specified subset of the final exams. Each final exam is a single time slot in length. The problem is to determine if these finals can be scheduled in only h time slots so that no student has two simultaneous exams. Formulate this problem as a language and show that it is NP-complete.

Bonus problems

1. Show that P is closed under the star operation.
2. A regular expression is *star-free* if it does not contain any star operations. Let $EQ_{SF-REX} = \{ \langle R, S \rangle \mid R \text{ and } S \text{ are equivalent star-free regular expressions} \}$. Show that $EQ_{SF-REX} \in \text{coNP}$. Why does the argument fail for general regular expressions?