Math 387

Homework 1

Due Friday, September 18

Practice exercises from the book

1.18, 1.20, 1.29, 1.46

Problems

1. For each of the following languages, give a regular expression that represents the language. In all cases $\Sigma = \{0, 1\}$.
   (a) $L = \{w| \; |w| \leq 5\}$
   (b) $L = \{w \mid w \text{ does not contain the substring } 001\}$
2. Show that the class of regular languages is close under intersection. That is, if A and B are both regular languages, then so is $A \cap B$.
3. For each of the following languages, prove either that it is regular or that it is not regular. In all cases $\Sigma = \{0, 1\}$.
   (a) $L = \{w \mid w \text{ contains an equal number of 0s and 1s}\}$
   (b) $L = \{1^k y \mid y \in \Sigma^*,\ k \geq 1, \text{ and } y \text{ contains at least } k \text{ 1s}\}$
   (c) $L = \{1^k y \mid y \in \Sigma^*,\ k \geq 1, \text{ and } y \text{ contains at most } k \text{ 1s}\}$
4. Consider the language $L = \{0^i 1^j 2^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.
   (a) Show that L is not regular.
   (b) Show that L does not look irregular as far as the pumping lemma goes. That is, give a pumping length p and show that L satisfies the conditions of the pumping lemma.
   (c) Explain why the two things you've shown above do not contradict.

Bonus problems

1. Let $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that if A and B are regular, then A/B is regular.
2. Our goal in this problem is to show that the representation of objects can affect whether or not a given set can be recognized by a machine. Consider a st A of natural numbers. Let $B_k(A)$ be the set of strings that represent numbers from A in base k (with no leading zeros). For example, if $A = \{3, 5\}$ then $B_2(A) = \{11, 101\}$ and $B_3(A) = \{10, 12\}$. We can think of $B_k(A)$ as a language with a k-symbol alphabet. Give a set A for which $B_2(A)$ is regular but $B_3(A)$ is not (and prove it).