Math 387

Homework 1

Due Friday, September 11

Practice exercises from the book

1.1, 1.2, 1.3, 1.6, 1.7, 1.11, 1.14

Problems

1. For each of the following languages, give a DFA that recognizes the language. In all cases $\Sigma = \{0, 1\}$.
   (a) $L = \{w \mid w$ is any string other than 11 or 111$\}$
   (b) $L = \{w \mid w$ contains the substring 001$\}$
   (c) $L = \{w \mid w$ has length at least 3 and has 0 for the third symbol$\}$
   (d) $L = \{w \mid w$ has a 1 in every odd position$\}$
   (e) $L = \{w \mid w$ a multiple of 3 1s or an even number of 0s$\}$
   (f) $L = \{w \mid w$, when thought of as a binary number, is a multiple of 7 $\}$
2. For each of the following languages, give a NFA that recognizes the language using no more than the listed number of states. In all cases $\Sigma = \{a, b, c\}$.
   (a) $L = \{\epsilon\}$, 1 state
   (b) $L = \{w \mid w$ ends in aa$\}$, 3 states
   (c) $L = \{w \mid w$ contains a multiple of 3 a's or a multiple of 4 b's$\}$, 8 states
   (d) $L = \{w \mid w$ ends in the first occurrence of some symbol$\}$, 5 states

Bonus problems

1. In class we showed that any n-state NFA can be converted to a 2^n-state DFA. Show that this bound is roughly tight. Specifically, show that for every n there exists a language that can be recognized with an $n+1$-state NFA but cannot be recognized by a DFA with fewer than 2^n states.