Math 387

Homework 4

## Due Monday, November 3

Practice exercises from the book

7.22, 7.25, 7.31, 7.34, 7.38

Problems

1. Take $DOUBLESAT = \{< \phi > \mid \phi$ is a boolean formula with at least two different satisfying assignments$\}$. Show that this language is NP-complete.

2. A *coloring* of a graph is an assignment of way of assigning a color to each vertex so that no two adjacent vertices have the same color. Take $3COLOR = \{< G > \mid G$ has a coloring that uses only three colors$\}$. Show that $3COLOR$ is NP-complete. (Hint: Use the following subgraphs.)

palette   variable   OR-gadget
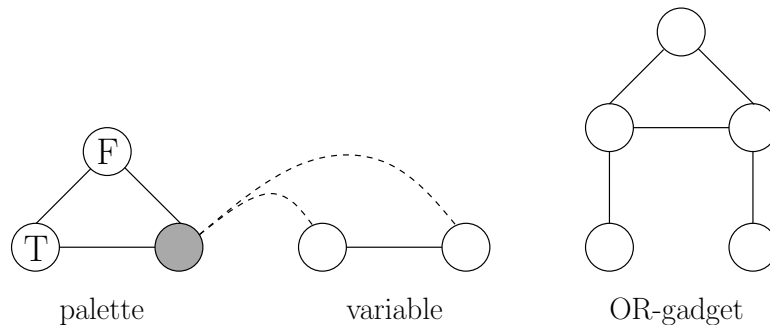
3. Let $SET\text{-}SPLITTING = \{< S, C > \mid S$ is a finite set and $C = \{C_1, \ldots, C_k\}$ is a collection of subsets of S such that each element of S can be colored red or blue so that each C_i has at least one element of each color$\}$. Show that this language is NP-complete.

4. Considering the following problem. You have a list of final exams and a list of students. Each student is taking some specified subset of the final exams. Each final exam is a single time slot in length. The problem is to determine if these finals can be scheduled in only h time slots so that no student has two simultaneous exams. Formulate this problem as a language and show that it is NP-complete.

Bonus problems

1. A regular expression is *star-free* if it does not contain any star operations. Let $EQ_{SF\text{-}REX} = \{< R, S > \mid R$ and S are equivalent star-free regular expressions$\}$. Show that $EQ_{SF\text{-}REX} \in$ coNP. Why does the argument fail for general regular expressions?