Math 387

Homework 4

Due Friday, October 17

Practice exercises from the book

7.1, 7.2, 7.7, 7.11, 7.13, 7.19

Problems

1. Take $MODEXP = \{< a, b, c, p > \mid a, b, c$ and p are binary integers and $a^b \equiv c \pmod{p}\}$. Show that $MODEXP \in P$. (Hint: The obvious algorithm doesn't run in exponential time. Try it first where b is a power of 2.)

2. Show that P is closed under union, concatenation, and complement.

3. Show that P is closed under the star operation. (This is hard. You will want to use dynamic programming, doing something that is in some ways similar to what we did in class to show that context-free languages were in P.)

4. Let $3COLOR$ be the set of 3-colorable graphs. That is, $3COLOR = \{< G > \mid G$ is a graph, and we can assign each node of G one of three colors such that no two nodes of G that are connected by an edge have the same color)$\}$. Show that $3COLOR \in NP$.

5. Show that if $P = NP$, then every langauge in P, except for $\emptyset$ and Σ^*, is NP-complete. Why do we need to make exceptions for $\emptyset$ and Σ^*?

Bonus problems

1. Write a computer program (in any langauge) that will print output exactly equal to the text of the program. (See 6.1 in the book for some thoughts that will help.)

2. Show that if $P = NP$ we can construct a polynomial-time algorithm to find a satisfying assignment for a given satisfiable boolean formula. (Note: If $P = NP$ that means that SAT is in P, but P as we have defined it contains languages, and machines give yes/no answers. What you are being asked to do here is create a machine that computes a function. You can think of this as a Turing machine that starts with the input on its tape, and then eventually halts with the ouptut of the function on its tape instead.)