Math 387

Homework 3

Due Friday, September 26

Practice exercises from the book

2.30, 2.42, 3.9, 3.22

Problems

1. For each of the following languages, prove either that it is context free or that it is not. In all cases $\Sigma = \{0, 1\}$.
   (a) $L = \{0^i 1^j \mid i \text{ is not a multiple of } j\ \}$
   (b) $A = \{w \mid w \text{ contains twice as many 0s as 1s}\}$
2. Consider the language A above consisting of all strings (with alphabet $\{0, 1\}$) that contain twice as many 0s as 1s (in any order).
   (a) Write an English-language description of a Turing machine for this language.
   (b) Draw the state diagram of the Turing Machine described above. (Some choices of answers for the previous part, while correct, will make this very hard. Feel free to alter your answer so that this part is easier, but make sure that this machine operates in a way consistent with the description above.)
   (c) Write the computation history of your machine on the string 01011.
3. 5-in-1 Turing machines are Turing machines that are allowed to write up to 5 symbols in any box (and with a transition function that takes into account the presence of up to 5 symbols). Show that 5-in-1 Turing machines are equal in power to regular Turing machines.

Bonus problems

1. With $\Sigma = \{0, 1, 2, 3\}$, take the language $L = \{w \mid w \text{ contains the same number of 0s as 1s and the same number of 2s as 3s}\}$. Prove that L is context free or that it is not.
2. Consider a Turing machine that cannot write over its input. That is, whatever length of string that the input is on cannot be changed, but can be read as normal (and the rest of the tape can be changed as normal). Show that Turing machines of this type can recognize only regular languages.