Math 387

Homework 2

Due Friday, September 19

Practice exercises from the book

1.29, 1.46, 2.1, 2.4, 2.5, 2.6, 2.9, 2.10, 2.15

Problems

1. For each of the following languages, prove either that it is regular or that it is not regular. In all cases $\Sigma = \{0, 1\}$.

    (a) $L = \{w \mid w$ contains an equal number of 0s and 1s$\}$

    (b) $L = \{1^k y \mid y \in \Sigma^*, k \geq 1,$ and y contains at least k 1s$\}$

    (c) $L = \{1^k y \mid y \in \Sigma^*, k \geq 1,$ and y contains at most k 1s$\}$

2. For each of the following languages, give a CFG that generates the language. In all cases $\Sigma = \{0, 1\}$.

    (a) $L = \{w \mid w$ contains at least three 1s$\}$.

    (b) $L = \{w \mid w$ has odd length and its middle symbol is 0$\}$.

    (c) $L = \{0^m 1^n \mid m \neq n\}$.

3. Draw the state diagram of a PDA that accepts each of the following languages. In all cases $\Sigma = \{0, 1\}$.

    (a) $L = \{0^m 1^n \mid m \neq n\}$.

    (b) $L = \{w \mid w$ has more 0s than 1s$\}$.

Bonus problems

1. Let G be the following CFG:

$$S \to \mathrm{a}S\mathrm{b} \mid \mathrm{b}Y \mid Y\mathrm{a}$$
$$Y \to \mathrm{b}Y \mid \mathrm{a}Y \mid \epsilon$$

Give a simple English description of the language of G. Use this description to give a CFG that recognizes the complement of that language.

2. $L = \{xy \mid |x| = |y|$ and $x \neq y\}$.

    (a) Give a CFG that generates the language L.

    (b) Give a PDA that accepts the language L.