Math 387

Homework 1

Due Friday, September 12

Practice exercises from the book

1.1, 1.2, 1.3, 1.6, 1.7, 1.11, 1.14, 1.18, 1.20

Problems

1. For each of the following languages, give a DFA that recognizes the language. In all cases $\Sigma = \{0, 1\}$.
   (a) $L = \{w \mid w \text{ is any string other than } 11 \text{ or } 111\}$
   (b) $L = \{w \mid w \text{ contains the substring } 001\}$
   (c) $L = \{w \mid w \text{ has length at least 3 and has 0 for the third symbol}\}$
   (d) $L = \{w \mid w, \text{ when thought of as a binary number, is a multiple of 7 }\}$
2. For each of the following languages, give a NFA that recognizes the language using no more than the listed number of states. In all cases $\Sigma = \{a, b, c\}$.
   (a) $L = a^*(bbaa^*)^*$, 3 states
   (b) $L = \{w \mid w \text{ ends in aa}\}$, 3 states
   (c) $L = \{w \mid w \text{ ends in the first occurrence of some symbol}\}$, 5 states
3. For each of the following languages, give a regular expression that represents the language. In all cases $\Sigma = \{0, 1\}$.
   (a) $L = \{w | \, |w| \leq 5\}$
   (b) $L = \{w \mid w \text{ does not contain the substring } 001\}$
4. Show that the class of regular languages is close under intersection. That is, if A and B are both regular languages, then so is $A \cap B$.

Bonus problems

1. In class we showed that any n-state NFA can be converted to a 2^n-state DFA. Show that this bound is roughly tight. Specifically, show that for every n there exists a language that can be recognized with an $n+1$-state NFA but cannot be recognized by a DFA with fewer than 2^n states.
2. Let $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that if A and B are regular, then A/B is regular.