Math 382

Homework 1

Due Friday, February 5

1. Prove that $125n^2 + 30 \in \Omega(n^2)$. (Do this directly from the definition of $\Omega(\cdot)$. Do not use results we proved in class about polynomials.)

2. Rank the following functions by order of growth. That is, create a list $f_1, f_2, \ldots$ such that $f_1 \in O(f_2), f_2 \in O(f_3), \ldots$. Circle in your list groups of functions that are equivalent (i.e., $f_i \in \Theta(f_{i+1})$).

$2^{\log_5(n)}$	n^2	$n!$	$(n+1)^2$	$(3/2)^n$
n^3	$\log_2^2(n)$	$35n^2 + 15$	$\log_2(\log_2(n))$	$n \cdot 2^n$
$\log_2(n)$	1	3^n	$(n+1)!$	$\sqrt{n}$
2^n	$\log_5(n)$	n^n	$\log_4(n^2)$	$n \log_2(n)$
$4n$	n^{100}	$n^3 + n^2$	2^{2n}	2^{n+3}

3. Let $f(n)$ and $g(n)$ be nonnegative functions. Prove or disprove each of the following statements:

   (a) $\max(f(n), g(n)) \in \Theta(f(n) + g(n))$ for all $f(n), g(n)$.

   (b) $f(n) + g(n) \in \Theta(f(n))$ for all $f(n)$ and $g(n) \in o(f(n))$.

   (c) $f(n) \in \Theta(f(n/2))$ for all $f(n)$.

   (d) $f(n)^2 \in \Theta(g(n)^2)$ for all $g(n)$ and $f(n) \in \Theta(g(n))$.

4. Prove that if $f(n)$ and $g(n)$ are nonnegative functions, then $f(n) \in o(g(n))$ if and only if $\lim_{n\to\infty} \left(\frac{f(n)}{g(n)}\right) = 0$.