Math 382

Homework 3

Due Wednesday, February 18

1. Below is pseudocode for a new sorting algorithm, NewSort. You should look over the code and make sure you understand how and why the algorithm works.

   (a) Prove that NewSort is indeed a correct sorting algorithm.

   (b) How long does NewSort take to run? Prove your answer.

```
Define NewSortHelper(A, start, end):
    if start = end then
        return
    if start+1 = end then
        if A[start] > A[end] then
            swap A[start], A[end]
        return
    t = ⌊(end−start+1)/3⌋
    NewSortHelper(A, start, end − t)
    NewSortHelper(A, start + t, end)
    NewSortHelper(A, start, end − t)
Define NewSort(A):
    NewSortHelper(A, 1, A.length)
```

2. When discussing mergesort, we gave an algorithm MERGE which took two sorted lists and returned a sorted list that contained the elements of both input lists. This algorithm took $O(n)$ time, where n was the total number of elements in the two lists combined. Now consider the case of merging k separate sorted lists, again with n total elements in all lists combined. Find an algorithm that runs in $O(n \lg k)$ time. (Hint: One option is to use a heap to help.) Show that this really is the runtime of your algorithm.

3. SUM2(S, t) is a a function that takes as input a set S of numbers and a target number t. The output should be a boolean equal to "true" if there are two (distinct) numbers in S whose sum is t and "false" otherwise. Find an algorithm for SUM2 and analyze it. (Any correct algorithm with a correct analysis receives some credit, but the faster it is the better.)