Theorem 1. The derivative of a function \( f \) at \( c \) is given by

\[
f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}
\]

provided this limit exists.

Proof. By definition, we know that

(1) \[
f'(c) = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}.
\]

Let \( x = c + \Delta x \). Then \( x \to c \) as \( \Delta x \to 0 \). So, replacing \( c + \Delta x \) by \( x \) and \( \Delta x \) by \( x - c \) in (1), we find that

\[
f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}
\]

as desired. \qed

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